Persistence of Temporary Employment: The Role of Training*

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Abstract

Temporary employment is widespread and persistent in advanced economies. The data show that: (i) the less-educated are over-represented in temporary jobs; (ii) although training accelerates the transition to permanent employment, training incidence is relatively low among temporary workers. To understand these facts, I build a search model with endogenous contract types and training decisions. In the model, firms hire the low-skilled as temporary workers to avoid the firing costs imposed on permanent contracts, and employers provide little training to temporary workers because the investment horizon is truncated. Consequently, the model endogenously reproduces the persistence of temporary employment, known as the "temporary job trap." I estimate the model on Korean labor market data. The results suggest that current policy depresses output 9.4% below its potential, with 43% of that gap attributable to the lack of training for temporary workers. Counterfactual policy experiments indicate that training subsidies can eliminate nearly 80% of the output loss.

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1 Introduction

Temporary employment refers to having a job that is not protected as much as a regular one (OECD, 2014). To prevent the excessive expansion of less or non-protected jobs, most developed countries restrict the use of temporary contracts by limiting, for example, their maximum duration, or the number of contract renewals.[1] Temporary employment, nonetheless, is widespread in developed countries (see Figure 1.(a)). In Spain and Korea, for instance, temporary workers accounted for 26.7 and 20.6 percent, respectively, of the total labor force in 2017 (OECD, 2018). Previous studies have widened our understanding of temporary employment.[2] At the cross-sectional level, those with less education are over-represented in temporary jobs (Charlot and Malherbet, 2013), and temporary workers receive relatively little on-the-job training (Cabrales et al., 2017). At the time-series level, the rate of conversion from temporary to permanent (regular) employment is limited (Berton et al., 2011), but on-the-job training accelerates the transition (European Commission, 2004).

Why does temporary employment arise and persist? I provide an answer to this question by focusing on human capital and its accumulation on the job. Despite existing evidence on the link between human capital and temporary employment, there is no unified structural framework that can endogenously reproduce the above-mentioned stylized facts. As a consequence, the current literature cannot explain the observed cross-sectional and dynamic patterns of temporary employment within a single framework. Furthermore, existing models cannot address policy-relevant questions, such as how training incidence and labor market mobility (in terms of contract types) interact with each other, and how their interaction can be affected by a change in labor market institutions. This paper advances our understanding by proposing an analytically tractable and empirically plausible framework.

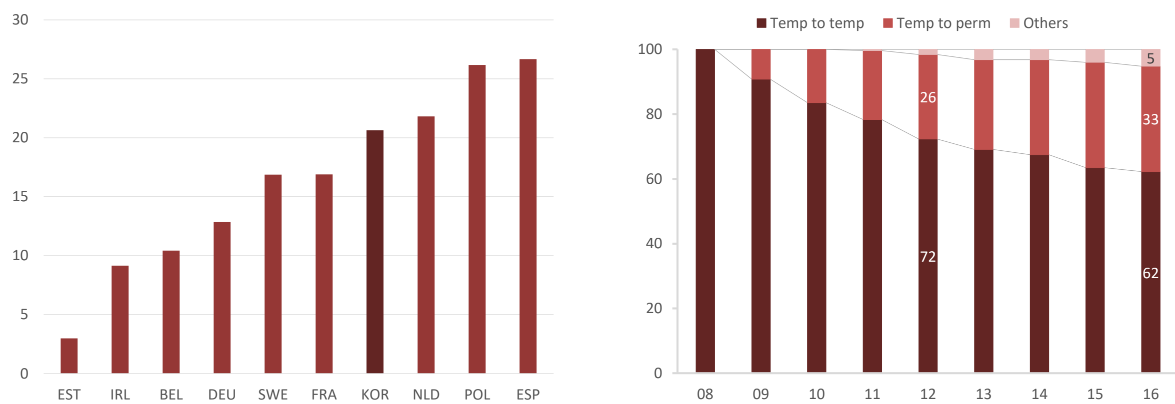
A novel feature of my model is endogenous contract-type and training decisions in the presence of different human capital levels. This key aspect allows me to make three contributions to the literature. First, I provide theoretical explanations for the sorting of the less-educated into temporary jobs, and the lower incidence of training among temporary workers. As a consequence, my model also illuminates the theoretical background behind the persistence in temporary employment, known as the "temporary job trap." Second, I investigate the welfare consequences of the existence and persistence of temporary employment in both qualitative and quantitative ways. I establish that the decentralized economy cannot attain the constrained social optimum in terms of total output, and quantify output loss by estimating the model on Korean labor market data. Third, I conduct policy experiments to examine possible sources of improvement in efficiency. The counterfactual analysis shows that training subsidies can lead to substantial output gains by inducing workers and firms to invest more in on-the-job training.

My model is built on the standard random-search framework (e.g. Mortensen and Pissarides, 1994). The economy consists of risk-neutral workers and firms. Workers (Firms) can be either employed or unemployed (filled or vacant). Search frictions exist (finding a job or a worker takes time), and thus, a worker-firm match creates a surplus which is shared between both parties. The match surplus is divided via Nash bargaining. Payoffs to both parties thus increase in match surplus, meaning that a larger match surplus is preferred by both worker and firm. An equilibrium is a labor market tightness (the ratio of vacancies to unemployment), which is pinned down by the free-entry condition (the expected net value of a firm at entry must be zero).

[1] Thus, temporary jobs typically last for a short period of time, which is the reason why they are called "temporary" while their regular counterparts "permanent."

[2] See Figure 1.(b), Figure 2, and Table 1 for a brief summary of the following stylized facts.

Figure 1: The incidence and persistence of temporary employment



Notes: (a) The figure is drawn based on the data from OECD (2018). The share is expressed as a percentage of dependent employees aged 15 or older. The definition of temporary employment may vary from country to country, and interested readers are referred to the original source available at `http://dx.doi.org/10.1787/888933779181` (last accessed on November 1, 2019). (b) I utilize data from the Korean Labor and Income Panel Study to illustrate the dynamics of temporary employment in Korea. All temporary workers in the baseline year (2008) are classified into three groups based on changes in their employment status. The group labeled "Temp to temp" consists of those who have never achieved permanent employment status since the baseline year. The group labeled "Temp to perm" is composed of those who have achieved permanent employment status *and* have not returned to temporary jobs since then. The group labeled "Others" includes those who have rotated between temporary and permanent jobs over the period considered.

I extend this standard model in three dimensions to study a trade-off between temporary and permanent contracts, and training decisions in the presence of such a trade-off. First, I introduce worker heterogeneity in human capital. Specifically, my model categorizes human capital into general and specific (Becker, 1964),[3] and workers differ in general human capital. However, they start a new job with the same match productivity (specific human capital). Firms are still identical, but the production technology is modified to be the product of two types of skills. Second, I formulate temporary and permanent employment, à la Cahuc et al. (2016). As in the standard model, all matches are subject to separation shocks. When hit by a separation shock, the destruction of a permanent match incurs firing costs (constant across all workers) while the destruction of a temporary match does not. Temporary contracts, however, last only for a specified period, resulting in a higher turnover rate. Third, I allow both general and specific human capital to be accumulated through training. Expecting improvement in general or specific ability, the worker-firm pair can allocate an exogenous amount of time to training. Contrary to Flinn et al. (2017), investment in training occurs at the extensive margin.[4] Since the worker and firm bargain over both contract-type and training, there are six options on the negotiation table in the end. Wages are set by Nash bargaining in all cases. Therefore, the worker and firm choose an option that maximizes total match surplus.

In this setting, my model endogenously generates the temporary job trap in an empirically consistent manner. Two mechanisms are combined. First, low-ability workers are sorted into temporary jobs because their ability is too low to justify firing costs imposed on permanent contracts. In my model, the worker-firm

[3]General human capital refers to a type of human capital that is equally productive in all jobs. Specific human capital, on the other hand, refers to a type of human capital that is productive in a given job but not in the other jobs.

[4]In other words, the worker and firm can choose either no training, general training, or specific training.

pair chooses a permanent contract if and only if the expected match surplus accruing beyond the duration limit of temporary contracts is greater than firing costs. The expected surplus arising from the continued match is increasing in the worker's general human capital, while firings costs are independent of it. Hence, there is a cutoff level of general human capital, and those below that cutoff become temporary workers. Second, the worker-firm pair on a temporary contract has little incentive to invest in training due to the worker's low ability and the short investment horizon. In my framework, it is assumed that training is more costly to those with lower ability (Autor, 2001). Because temporary contracts are held by the low-skilled as claimed above, little investment in training of temporary worker-firm pairs arises (a direct selection effect). Furthermore, temporary contracts selected by low-ability workers reduce the net benefits of training as well since the investment horizon is truncated by the fixed duration of contracts (an indirect selection effect). To summarize, these direct and indirect selection effects, combined with the sorting mechanism of low-ability workers into temporary jobs, endogenously reproduce the persistence in temporary employment.

To derive quantitative implications of my model, I estimate it on Korean labor market data using the method of simulated moments. The Korean labor market is a suitable laboratory for quantitative analysis because it is facing challenges from the rise of temporary employment (Dao et al., 2014; Jones and Fukawa, 2016; Schauer, 2018). The estimated model captures the prominent features of the data well. In particular, the persistence of temporary employment (the "transition" rates from temporary to temporary contracts) is well reproduced by the model although it is not explicitly targeted in the estimation. The estimated model allows me to (i) document scarring effects of entering the labor market as a temporary worker, (ii) quantify the output loss and attribute it to distinct factors in the model, and (iii) conduct counterfactual analysis.

First, using both real and simulated data, I detect the potential scarring effects of starting a career with a temporary contract on current labor market outcomes such as training and wages. Having a temporary contract in one's first job leads to a lower likelihood of receiving any training, and lower wages in one's current job. In addition, I observe that these scarring effects are overestimated in the simulated data. Since my model represents the extreme case of a labor market where on-the-job training is the only stepping stone towards permanent employment, the overestimation *per se* is not surprising. Nonetheless, it is remarkable that the estimates from the actual data are quite close to their simulated counterparts: the scarring effects on training and wages, estimated from the real data, amount to nearly four-fifths and three-fifths, respectively, of their simulated counterparts. This analysis, therefore, provides new evidence that not only do training and wages exhibit a long-run response to first contract type, but also that their response is sizable.

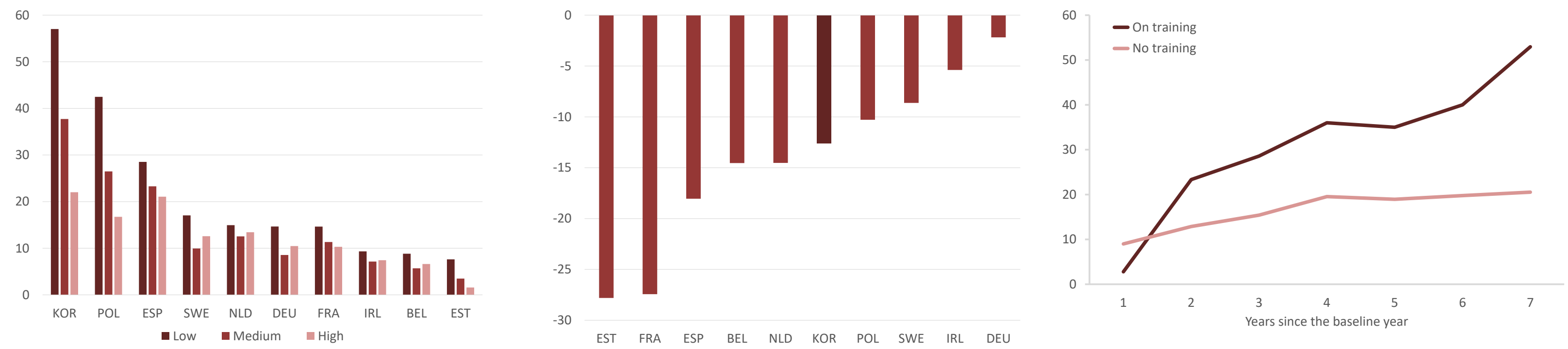
Second, I measure how far the decentralized economy is away from its potential. I consider the constrained planner who, respecting current labor market frictions and institutions, chooses contract and training types, along with a labor market tightness, to maximize total output less vacancy and training costs. Theoretically, the constrained planner's solution does not coincide with the equilibrium outcome for two reasons. First, as long as vacancy costs are sufficiently small, the worker-firm pair overvalues permanent jobs (Jarosch, 2015), leading to inefficient contact-type choices. When assessing the value of a potential match, the worker-firm pair does not fully internalize the gains from future matches to be formed as the worker becomes unemployed; they ignore the gains that accrue to the worker's future employers. Permanent jobs are accordingly overvalued relative to the efficient benchmark, unless creating a vacancy is too costly. Second, the decentralized training decisions are suboptimal due to the positive externalities associated with general training (Acemoglu, 1997). The benefits from general training are shared by the worker's future employers although they pay nothing for their benefits. Consequently, either specific or no training can be "wrongly" chosen over general training in the decentralized economy. The quantitative analysis complements the theoretical arguments. The current labor market regime depresses output 9.4 percent below its potential. The gap between the *status quo* and the ideal is mostly caused by the lack of general training for temporary workers (43%). Specific training "wrongly" chosen by permanent workers (39%) and too many permanent jobs (18%) explain the rest.

Third, I explore the possibilities of efficiency improvement by conducting counterfactual policy experiments. I consider two policy options: subsidizing permanent employment, and subsidizing on-the-job training (both financed via lump-sum taxes on employed workers). Contrary to the implication of this study, temporary employment has been mostly regarded as undesirable. Thus, policies aimed at encouraging permanent employment have been widely adopted and studied (e.g. Arranz et al., 2013; García-Pérez and Osuna, 2014). Training subsidies, on the other hand, despite their general effectiveness (Heckman, 2000; Blundell et al., 2019), have been largely overlooked in the temporary employment literature. Consequently, little is known about the effectiveness of training subsidies in the presence of temporary contracts. The experiments with the two policy regimes demonstrate the relative effectiveness of training subsidies in improving social efficiency. Training subsidies can eliminate nearly 80 percent of the output loss, whereas only 15 percent of it can be removed by permanent-job subsidies. The resulting implication is that the extent to which permanent-job subsidies indirectly promote training is quite limited, and thus, subsidizing permanent jobs is not effective to recover the efficiency loss which is primarily caused by suboptimal training.

The rest of the paper is organized as follows. The next section introduces and investigates a benchmark model with two types of contracts available. On-the-job training is incorporated to the model in Section 3. Section 4 discusses the estimation strategy and results, examining the model fit to the data. Counterfactual experiments are designed and implemented in Section 5. Section 6 concludes with remarks on future work.

Figure 2: Features of temporary employment

(a) Temporary employment by education level | (b) Temporary employment and training | (c) Transition to permanent employment



Notes: (a) The figure is drawn based on the data collected by the OECD on people aged 25-54 in 2011-12. The classification of the level of education is different country by country, and readers interested in details are referred to the original source: `http://dx.doi.org/10.1787/888933132659`. The original source does not include relevant data from Korea, for which alternative data provided by the National Statistical Office of Korea are used. (b) The figure indicates the estimated percentage difference in the probability of receiving on-the-job training between temporary and permanent workers in 2012. The original data collected by the OECD can be accessed at `http://dx.doi.org/10.1787/888933132811`, in which Korea is reported to have minus 4.0 percentage points. However, the number is not statistically significant at conventional significance levels, and thus, an alternative estimate (minus 12.6 percentage points) suggested by the National Statistical Office of Korea is reported. All online references were accessed on November 1, 2019. (c) The figure shows the conversion rate to permanent employment for two groups of Korean temporary workers: those receiving on-the-job training at least once for the past three years, and those never receiving over the same period. The horizontal axis corresponds to years since the baseline year (2010) when workers were grouped according to their training experience. For details on the data (the KLIPS) used for the plot, see Section 4.1.

Table 1: A snapshot of the reduced-form analysis results

(a) Determinants of temporary employment

	(1)	(5)
Level of edu.		
Secondary	−0.488***	−0.293***
	(0.063)	(0.075)
Tertiary	−1.133***	−0.734***
	(0.071)	(0.096)
Char. of workers	N	Y
Char. of jobs	N	Y

(b) Determinants of receiving training

	(1)	(3)
Contract type		
Temporary	−0.494***	−0.295***
	(0.080)	(0.084)
Char. of workers	N	Y
Char. of jobs	N	N

(c) Determinant of conversion to perm

	(4)	(6)
Training exp.		
(extensive)	0.189**	0.255**
	(0.092)	(0.112)
Char. of workers	Y	Y
Char. of jobs	Y	Y

Notes: The tables summarize the results obtained from the reduced-form analysis (using probit models) reported in Appendix B.2, and thus, the column numbers in (a)–(c) correspond to those in Tables 8–10, respectively. For complete results and detailed commentaries, see Tables 8–10 in Appendix B.2 and the notes therein.

2 The Benchmark Model

In order to study the underlying trade-off between permanent and temporary contracts, this section develops a simple framework without human capital accumulation on the job. The model is built based on a standard random search-and-matching model (e.g. Mortensen and Pissarides, 1994). However, for the sake of transparency, I exclude productivity shocks that may reflect fluctuations in the demand for firms' products; instead, two types of employment contracts (permanent versus temporary) are introduced in the labor market, as in Berton and Garibaldi (2012), Lepage-Saucier et al. (2013), and Cahuc et al. (2016). The model presented in the current section will be extended to include on-the-job training in Section 3.

2.1 Setup

Agents One side of the labor market is constituted by infinitely lived workers, whose measure is normalized to 1. Workers differ in the level of *general* human capital $z \in \mathbb{Z}$, which is constant over the lifetime in the current setup. Let $L(\cdot)$ be the cumulative distribution function of z, and $\ell(\cdot)$ be its density. It is assumed that $L(\cdot)$ has no mass points, and $\ell(z) > 0$ for all $z \in \mathbb{Z}$. The other side of the labor market is populated by homogeneous firms. In particular, all firms are identical in terms of initial productivity (the level of specific human capital with which a worker starts a new job) and security (the exogenous separation rate). Each firm is able to employ at most one worker,[5] and each worker cannot be employed by more than one firm. Workers and firms are risk-neutral, and they have a common discount rate $r > 0$.

Search and matching Time is continuous. Only and all unemployed workers and unfilled jobs engage in search activities. Search is random, and governed by a constant return to scale (CRS) matching function $M : [0, 1] \times \mathbb{R}_+ \to [0, 1]$. Let U be the number of unemployed workers, and V be the number of unfilled jobs (vacancies). Using the CRS property of $M(\cdot, \cdot)$, I denote the job-finding rate of unemployed workers by $p(\theta) := M(U, V)/U$, and the worker-meeting rate of vacancies by $q(\theta) := M(U, V)/V$, where $\theta := V/U$ is the labor market tightness to be determined in equilibrium. Standard regularity conditions on $p(\cdot)$ and $q(\cdot)$ apply so that $q'(\theta) < 0 < p'(\theta)$, $\lim_{\theta\to 0} p(\theta) = \lim_{\theta\to\infty} q(\theta) = 0$, and $\lim_{\theta\to\infty} p(\theta) = \lim_{\theta\to 0} q(\theta) = 1$.[6]

Once an unemployed worker and a vacant job meet each other, they jointly decide whether to form a match or not, based on the expected surplus accruing from the match. When calculating the match surplus, they weigh two alternatives: permanent versus temporary contracts. If the permanent contract is chosen, the employment relationship persists until the arrival of the (exogenous) separation shock which follows Poisson process with rate $\delta > 0$. Once the permanent match is hit by the separation shock, the worker becomes unemployed and immediately starts searching for a new job while the job, after paying red-tape firing costs $\kappa > 0$,[7] disappears. If the temporary contract is opted for, on the other hand, it is stipulated that the employment relationship is terminated (without incurring any red-tape firing costs to the firm) when the worker's job tenure λ reaches the predetermined employment duration $\Lambda > 0$. Note that temporary matches are also subject to the separation shock, but temporary jobs are exempted from paying κ in case of its arrival.

[5] Therefore, "firm" and "job" are interchangeably used in this paper.

[6] It is also worth noting that the CRS property implies $p(\theta) = M_1(1, \theta) + \theta p'(\theta)$, which will be recalled when establishing the existence and uniqueness of stationary equilibrium in Section 2.2.

[7] As explicitly described in the text, κ is bureaucratic costs, meaning that it is not severance payments (i.e. transfers from the firm to the worker). Thus, a match surplus is affected by κ, along the same lines as Cahuc et al. (2016).

Obviously, the surplus (dis)advantage of one contract type relative to another type depends on the worker's general human capital level, and the worker-job pair chooses the contract type that maximizes the match surplus. Under both permanent and temporary contracts, wages are determined by Nash bargaining with workers' bargaining power $\beta \in (0, 1)$.[8] Therefore, maximizing the match surplus is equivalent to maximizing either the worker's or the firm's surplus, meaning that disagreement over the contract type does not arise. Finally, a match is consummated if and only if the maximized surplus is nonnegative,[9] and neither the contract type nor the wage is renegotiated.

Discussion on modeling the temporary contract Before formulating value functions based on the current environment, I briefly discuss characteristics of the temporary contract designed in the model. First, all surviving temporary contracts are terminated at Λ, meaning that the temporary-to-permanent conversion within the same firm is not allowed in the model. I abstract from such a possibility because the transition from temporary to permanent contracts within the same firm is rarely observed in the data. Second, for the sake of simplicity, the renewal of temporary contracts is not considered in the model. In quantitative analysis, I lessen this gap between the model and reality by treating Λ as the policy parameter that governs not only the maximum duration of temporary contracts but also the maximum number of contract renewals, and estimating it. Lastly, if the separation shock arrives before Λ, the temporary match is immediately destroyed at no cost in the model. However, the temporary contract triggers red-tape firing costs as well in practice (albeit less burdensome relative to the permanent contract), and thus, I emphasize that κ in fact stands for the firing-cost *gap* between the two types of contracts, as in Garda (2013).

Value functions for workers For a worker of type $z \in \mathbb{Z}$, let $W_u(z)$ denote the value of unemployment, $W_p(z)$ the value of starting a permanent contract, and $W_t(z)$ the value of starting a temporary contract. Assuming that all meetings lead to matches (which will be confirmed later) allows me to write worker z's flow Bellman equation when unemployed as follows:

$$rW_u(z) = bz + p(\theta)\left[W(z) - W_u(z)\right]. \tag{1}$$

When unemployed, worker z receives $bz > 0$, instantaneous unemployment benefit depending on his general human capital level. The matching technology implies that the worker meets a firm at rate $p(\theta)$. When the search is successful, the worker starts either a permanent or a temporary job to enjoy welfare gain $W(z) - W_u(z)$, where $W(z) := \max\{W_p(z), W_t(z)\}$ is the value of employment for worker z.

In the case of worker z employed at a firm under the permanent contract, the flow Bellman equation can be written as

$$rW_p(z) = w_p(z) + \delta\left[W_u(z) - W_p(z)\right]. \tag{2}$$

If worker z is permanently employed, he receives flow wage $w_p(z)$ that depends on the worker's type and remains constant for the life of the match. The permanent job can be destroyed at rate δ. In such a case, the worker experiences welfare loss $W_u(z) - W_p(z)$.

[8] In the benchmark model, it is assumed that temporary workers have the same bargaining power as permanent workers do, which will be relaxed in the quantitative analysis.

[9] That is, some meetings that are expected to yield a negative surplus do not result in matches, and unemployed workers and vacant jobs continue their search process. However, as discussed in detail later, all meetings in fact lead to matches in the current setup because the temporary contract ensures a positive surplus for all matches.

The flow value to worker z of starting a temporary job is determined by the following flow Bellman equation:

$$rW_t(z) = w_t(z) + \delta\left[W_u(z) - W_t(z)\right] + \mathrm{e}^{-(r+\delta)\Lambda}\left[rW_u(z) - w_t(z)\right]. \tag{3}$$

A type-z temporary worker receives wage $w_t(z)$ that depends on his type and remains the same for the duration of the contract. The temporary job is also subject to a separation shock so that it may be terminated before the specified termination date. The temporary job that has survived until Λ is inevitably destroyed, an event that occurs with probability $\mathrm{e}^{-\delta\Lambda} \in (0, 1)$. The change of the (discounted) flow value in that case is reflected in the last term on the right-hand side of (3).[10] Note that the last term converges to zero as Λ approaches infinity.

Value functions for firms Let Π denote the value of a vacant firm, $\Pi_p(z)$ the value of a firm starting a permanent contract with worker $z \in \mathbb{Z}$, and $\Pi_t(z)$ the value of a firm starting a temporary contract with worker $z \in \mathbb{Z}$. Let $u(z)$ denote the mass of unemployed type-z workers.[11] Assuming again for the moment that all meetings result in either permanent or temporary matches, I can write the flow Bellman equation describing Π as follows:

$$r\Pi = -c + \int_{z\in\mathbb{Z}} q(\theta)\frac{u(z)}{U}\left[\Pi(z) - \Pi\right] \mathrm{d}z. \tag{4}$$

A vacant firm pays an instantaneous cost $c > 0$ to maintain its vacancy. The vacant firm meets a worker of type z at rate $q(\theta)\frac{u(z)}{U}$, the product of the rate of meeting a worker of any type, $q(\theta)$, and the probability that this worker is of type z, $\frac{u(z)}{U}$. A successful search and the consequent match with worker z delivers welfare gain $\Pi(z) - \Pi$ to the firm, where $\Pi(z) := \max\{\Pi_p(z), \Pi_t(z)\}$ is the value of employing worker z. In what follows, I impose a free-entry condition, which requires the expected gain from the search to be equal to the cost of the search, namely, $\Pi = 0$.

The flow Bellman equation that determines the value of a firm starting a permanent contract with worker z reads

$$r\Pi_p(z) = yz - w_p(z) + \delta\left[\Pi - \kappa - \Pi_p(z)\right]. \tag{5}$$

A firm permanently matched with worker z earns flow profits $yz - w_p(z)$ per unit of time, where $y > 0$. Note that the arrival of a separation shock causes welfare loss $\Pi - \kappa - \Pi_p(z)$ to the firm, reflecting that the firm has to pay firing costs κ in case of match destruction.

If a firm decides to start a temporary contract with worker z, the corresponding value can be represented by the following flow Bellman equation:

$$r\Pi_t(z) = yz - w_t(z) + \delta\left[\Pi - \Pi_t(z)\right] + \mathrm{e}^{-(r+\delta)\Lambda}\left[r\Pi - yz + w_t(z)\right]. \tag{6}$$

The interpretation of (6) is straightforward (and thus, omitted), but it is worthwhile to mention that in (6) the firm temporarily employing worker z pays no firing costs under any circumstances.

[10] Readers who are interested in how to arrive at the above Bellman equations are referred to Appendix A.1.

[11] Thus, $U = \int_{z\in\mathbb{Z}} u(z)\,\mathrm{d}z$ by definition.

Surplus For a given worker $z \in \mathbb{Z}$, the surplus of starting a permanent contract, denoted by $S_p(z)$, and the surplus of starting a temporary contract, denoted by $S_t(z)$, are defined as

$$S_p(z) := W_p(z) - W_u(z) + \Pi_p(z),$$
$$S_t(z) := W_t(z) - W_u(z) + \Pi_t(z),$$

respectively.[12] Let $S(z) := \max\{S_p(z), S_t(z)\}$ be the surplus accruing from the match. Then a closed-form expression for $S(z)$ can be obtained by a simple four-step procedure. First, the assumption that wages are determined by Nash bargaining over $S(z)$ implies $W(z) - W_u(z) = \beta S(z)$, allowing me to rewrite (1) as

$$rW_u(z) = bz + p(\theta)\beta S(z). \tag{7}$$

Second, one can use (2), (3), (5), and (6) (with the definitions of $S_p(z)$ and $S_t(z)$) to arrive at

$$S_p(z) = \frac{yz - rW_u(z) - \delta\kappa}{r + \delta}, \tag{8}$$

$$S_t(z) = [1 - \mathrm{e}^{-(r+\delta)\Lambda}]\frac{yz - rW_u(z)}{r + \delta}. \tag{9}$$

Third, $rW_u(z)$ in (8) and (9) can be replaced with the right-hand side of (7), which results in

$$(r + \delta)S_p(z) = yz - bz - p(\theta)\beta S(z) - \delta\kappa, \tag{10}$$

$$(r + \delta)S_t(z) = [yz - bz - p(\theta)\beta S(z)]\,[1 - \mathrm{e}^{-(r+\delta)\Lambda}]. \tag{11}$$

Lastly, one can simultaneously solve (10) and (11) for two unknowns $S_p(z)$ and $S_t(z)$ to derive[13]

$$S(z) = \begin{cases} S_t(z) = \dfrac{[1 - \mathrm{e}^{-(r+\delta)\Lambda}](y - b)}{r + \delta + [1 - \mathrm{e}^{-(r+\delta)\Lambda}]p(\theta)\beta} z & \text{if } z < z_r, \\ S_p(z) = \dfrac{y - b}{r + \delta + p(\theta)\beta} z - \dfrac{\delta\kappa}{r + \delta + p(\theta)\beta} & \text{if } z \geq z_r, \end{cases} \tag{12}$$

where z_r denotes the marginal worker type who is indifferent between starting a permanent or temporary job (namely, $S_p(z_r) = S_t(z_r)$; see Figure 3.(a) for a numerical example), and is given by

$$z_r = \frac{r + \delta + [1 - \mathrm{e}^{-(r+\delta)\Lambda}]p(\theta)\beta}{\mathrm{e}^{-(r+\delta)\Lambda}(r + \delta)(y - b)}\delta\kappa. \tag{13}$$

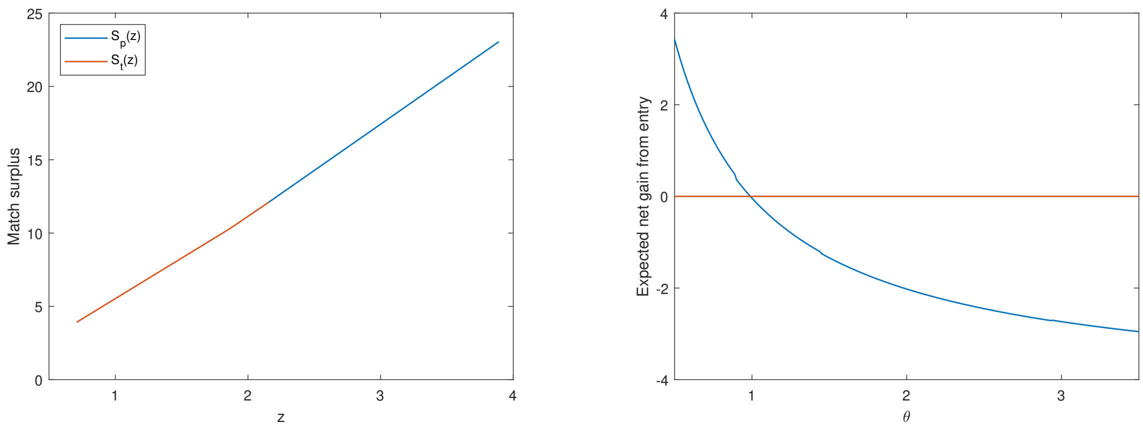
Discussion on $S(z)$ and z_r As indicated in the notation, I have derived the explicit forms of $S(z)$ and z_r by treating the labor market tightness $\theta \in (0, \infty)$ as exogenous. Before closing the model by utilizing the free-entry condition, I briefly present some properties of $S(z)$ and z_r (in the partial equilibrium environment) which are useful for later analysis. From now on, we suppose that $\mathbb{Z} = [\underline{z}, \overline{z}]$, where $\underline{z} > 0$ and $\overline{z} < \infty$.

[12] Note that the free-entry condition is already embedded in the definitions of $S_p(z)$ and $S_t(z)$.

[13] Explicit expressions for $S_p(z)$ and $S_t(z)$ are presented in more detail in Appendix A.2.

Figure 3: A graphical illustration of the benchmark model

(a) Surplus functions (b) The free-entry condition



Notes: Both (a) and (b) are drawn based on the model specification described in Section 4.2 and the estimated parameter values reported in Table 2, with the modification that the values of ρ, ϕ_s, and ϕ_g are all set to zero. Under the current parameter setting, $\theta = 0.990$ uniquely satisfies the free-entry condition (19).

(P1) *Positivity of $S(z)$.*

Provided that $y > b$ (i.e. market production is more efficient than home production), $S_t(z)$ (and thus, $S(z)$) is strictly positive for all $z \in \mathbb{Z}$. Accordingly, as long as $y > b$, all contacts between workers and firms lead to matches, as previously assumed. This is, of course, because temporary contracts incur no firing costs.

(P2) *Existence and uniqueness of z_r.*

With all parameter values fixed, z_r becomes a function of $\theta \in (0, \infty)$. Since $p'(\theta) > 0$ by assumption, $z_r(\theta)$ is increasing in θ, with its range bounded by $\lim_{\theta\to0} z_r(\theta)$ and $\lim_{\theta\to\infty} z_r(\theta)$. In order to avoid less interesting cases where permanent and temporary contracts do not coexist in equilibrium, I assume in what follows that $\mathbb{Z}$ is chosen such that $\underline{z} < \lim_{\theta\to0} z_r(\theta)$ and $\lim_{\theta\to\infty} z_r(\theta) < \overline{z}$. Meanwhile, both $S_p(z)$ and $S_t(z)$ are continuous piecewise linear functions in z, with a kink at $z = z_r$. As the slope of $S_p(z)$ is steeper than that of $S_t(z)$ for all $z \in \mathbb{Z}$ (see Appendix A.2), the marginal type z_r is unique as long as it exists.

(P3) *Trade-off between permanent and temporary contracts.*

From the explicit expressions for $S_p(z)$ and $S_t(z)$ (see Appendix A.2), it follows that $S_p(z) \gtreqless S_t(z)$ if and only if

$$\underbrace{\mathrm{e}^{-(r+\delta)\Lambda}\left[\frac{y-b}{r+\delta+[1-\mathrm{e}^{-(r+\delta)\Lambda}]p(\theta)\beta}\right]z}_{\text{relative costs of temporary contracts due to }\Lambda} \gtreqless \underbrace{\left(\frac{\delta}{r+\delta}\right)\kappa}_{\text{relative costs of permanent contracts due to }\kappa}. \tag{14}$$

The temporary job that has survived until Λ (an event occurring with probability $\mathrm{e}^{-\delta\Lambda}$) is unavoidably destroyed at Λ, which causes a surplus loss that amounts to $[r+\delta+(1-\mathrm{e}^{-(r+\delta)\Lambda})p(\theta)\beta]^{-1}(y-b)z$. The left-hand side of (14) exactly represents this amount of loss due to Λ, with discounting applied. On the other hand, the right-hand side of (14) stands for the discounted expected firing costs associated with permanent contracts.[14] Notice that the left-hand side is increasing in z whereas the right-hand side is independent of z, which reconfirms the uniqueness of z_r. It is also noteworthy that the trade-off between the two types of contracts diminishes as κ and Λ approach zero and infinity, respectively.

(P4) *Independence of S_r with respect to θ.*
Let S_r denote the total surplus of the match formed by worker z_r (i.e. $S_r := S(z_r)$). From (12) and (13) above, S_r is simply given by

$$S_r = \frac{1 - \mathrm{e}^{-(r+\delta)\Lambda}}{\mathrm{e}^{-(r+\delta)\Lambda}} \left(\frac{\delta}{r + \delta} \right) \kappa, \tag{15}$$

which is independent of the labor market tightness θ. An intuitive explanation is as follows. A rise in θ (and thus, $p(\theta)$) leads not only to a decrease in the slope of $S(z)$ (because of the increased opportunity costs of forming a match) but also to an increase in z_r (due to the increased value of being unemployed). However, the effect of the former is exactly offset by that of the latter, and consequently, the total surplus associated with the marginal worker type remains the same for all possible values of θ.

(P5) *Exogeneity of Λ.*
Some readers may ask whether the duration of a temporary contract can be endogenized, as in Cahuc et al. (2016). In order to inspect this possibility, one can differentiate $S_t(z)$ in (12) with respect to Λ to obtain

$$\frac{\partial S_t(z; \Lambda)}{\partial \Lambda} = \mathrm{e}^{-(r+\delta)\Lambda} \left[\frac{r + \delta}{r + \delta + [1 - \mathrm{e}^{-(r+\delta)\Lambda}] p(\theta) \beta} \right]^2 (y - b) z,$$

which is strictly positive as long as $y > b$. This observation implies that any worker-firm pair who are willing to form a temporary match would choose Λ if they are allowed to choose the duration of their temporary contract from the interval $[0, \Lambda]$. Therefore, although Λ is treated as exogenous throughout the paper, it can be regarded as an optimal choice made by agents.

2.2 Stationary Equilibrium

Flow equations As a last step to define a stationary equilibrium and study its properties, I examine the stationary distribution of workers. In a stationary equilibrium (to be formally defined below), the outflow from unemployment must equal the inflow into unemployment for each type of worker. More precisely, the following flow equation must hold for all $z \in \mathbb{Z}$:

$$\underbrace{p(\theta) u(z)}_{\text{outflow}} = \underbrace{\delta \left[\ell(z) - u(z) \right] + \mathbb{1}_{\{z < z_r\}} \left[\mathrm{e}^{-\delta \Lambda} p(\theta) u(z) \right]}_{\text{inflow into unemployment}}, \tag{16}$$

where $\mathbb{1}_{\{z<z_r\}}$ is an indicator function that takes the value one if $z < z_r$ and zero otherwise. As search is random, the flow of matches formed by job seekers of type z is simply given by $p(\theta)u(z)$. Meanwhile, the mass of employed type-z workers is $\ell(z) - u(z)$, and their jobs are destroyed with flow probability δ, as represented by the first term on the right-hand side of (16). The second term corresponds to the case where

[14]Indeed, recall that the separation occurrence follows the Poisson process to see $\int_0^\infty (\mathrm{e}^{-rt}\kappa)\delta \mathrm{e}^{-\delta t}\,\mathrm{d}t = \delta\kappa/(r + \delta)$.

temporary workers whose tenure reaches Λ go back to unemployment due to the nature of the temporary contract.[15]

Letting $u_t := \frac{\delta}{(1-\mathrm{e}^{-\delta\Lambda})p(\theta)+\delta}$ and $u_p := \frac{\delta}{p(\theta)+\delta}$, (16) can be equivalently rearranged as follows:

$$u(z) = \begin{cases} u_t \ell(z) & \text{if } z < z_r, \\ u_p \ell(z) & \text{if } z \geq z_r, \end{cases} \tag{17}$$

in which $u_t > u_p$, reflecting lower job security of temporary workers (relative to their permanent counterparts). As $\int_{z\in\mathbb{Z}} \ell(z)\,\mathrm{d}z = 1$ by assumption, integrating the both sides of (17) over $\mathbb{Z}$ yields the steady state unemployment rate, namely,

$$U = u_t L_r + u_p\left(1 - L_r\right), \tag{18}$$

where $L_r := \int_{z<z_r} \ell(z)\,\mathrm{d}z$ is the total mass of workers of type $z < z_r$.

Stationary equilibrium A stationary equilibrium for this economy is defined in a standard fashion. An equilibrium is a labor market tightness θ that satisfies (4), with the free-entry condition imposed, as formally stated in the following definition.[16]

Definition 1 (Equilibrium in the benchmark model)**.** A *stationary equilibrium* is a labor market tightness $\theta \in (0, \infty)$ satisfying the following free-entry condition:

$$c = \int_{z\in\mathbb{Z}} q(\theta)\frac{u(z;\theta)}{U(\theta)}(1-\beta)S(z;\theta)\,\mathrm{d}z, \tag{19}$$

where $S(z;\theta)$, $u(z;\theta)$, and $U(\theta)$ are given by (12), (17), and (18), respectively.

See Figure 3.(b) for a numerical example of the equilibrium.

The next goal is to establish the existence and uniqueness of the equilibrium. I introduce mild assumptions on the model parameters. First, to achieve the existence, in addition to $b < y$ which is motivated by (P1), I impose a sufficient condition ensuring that the cost of posting a vacancy is small enough so that each vacancy can expect a positive net gain from searching when search frictions for unfilled jobs are absent (that is, when $q(\theta) = 1$). Second, in order to prove the uniqueness result, I lay down two additional sufficient conditions, under which the cost of posting a vacancy is sufficiently large (compared to the firm's share of the surplus from a match associated with the marginal worker type), and the matching rate for a vacancy is always more elastic (with respect to θ) than the matching rate for a worker. Consequently, the decreasing monotonicity of the firm's expected net benefit from creating a vacancy (as a function of θ) is guaranteed.

[15] Notice that only a fraction $\mathrm{e}^{-\delta\Lambda}$ of newly-formed temporary matches, the flow of which is $p(\theta)u(z)$, will survive until Λ.

[16] From now on, for the sake of clarity, I indicate whether an expression depends on θ or not by adding θ to the notation when the expression is affected by it: e.g. $S(z;\theta)$, $u(z;\theta)$, $U(\theta)$, etc.

Proposition 1 (Existence and uniqueness of the equilibrium)**.**

(a) *A stationary equilibrium exists under the following conditions:*

(E1) $b < y$,

(E2) $c < \int_{z\in\mathbb{Z}} \ell(z)(1-\beta)S(z;0)\,\mathrm{d}z$, *with* $S(z;0) := \lim_{\theta\to 0} S(z;\theta)$.

(b) *Furthermore, the stationary equilibrium is unique if, in addition to* (E1) *and* (E2)*, the following additional conditions are satisfied:*

(U1) $(1-\beta)S_r \le c$, *where* S_r *is given by (15),*

(U2) $\left|\frac{\theta p'(\theta)}{p(\theta)}\right| \le \left|\frac{\theta q'(\theta)}{q(\theta)}\right|$ *for all* $\theta \in (0,\infty)$.

Proof. See Appendix A.3. □

Two remarks on the conditions appearing in Proposition 1 are in order. First, (U1) and (E2) provide lower and upper bounds, respectively, for the cost of posting a vacancy, and one may ask whether these two conditions are compatible with each other, namely, whether the following inequality holds in general:

$$(1-\beta)S_r < \int_{z\in\mathbb{Z}} \ell(z)(1-\beta)S(z;0)\,\mathrm{d}z. \tag{20}$$

Since it cannot be shown that (20) holds true without any restrictions on $\mathbb{Z}$ and $\ell(\cdot)$, I demonstrate the compatibility by presenting a simple example that requires only minimal assumptions on $\mathbb{Z}$ and $\ell(\cdot)$ in Appendix A.4. Second, (U2) is not too restrictive for the purpose of quantitative analysis. For instance, one may employ a Cobb-Douglas matching function of the form $M(U,V) = U^{1-\eta}V^{\eta}$ with $\eta \in (0,1)$ to have

$$\left|\frac{\theta p'(\theta)}{p(\theta)}\right| = \eta,$$
$$\left|\frac{\theta q'(\theta)}{q(\theta)}\right| = 1-\eta,$$

meaning that (U2) is satisfied as long as $\eta \le 0.5$.[17] With this observation in hand, it is worth mentioning that an elasticity of 0.5 ($\eta = 0.5$) is commonly used in the literature (see Petrongolo and Pissarides, 2001).[18]

Comparative statics Based on the existence and uniqueness results above, I study how the stationary equilibrium responds to a variation in two policy parameters κ and Λ.[19] First, a rise in the firing costs associated with permanent contracts affects the right-hand side of (19), the firm's expected benefit from creating a vacancy, in three distinct ways:

(K1) A *negative* effect of a decrease in the surplus of forming a permanent match;

(K2) A *positive* effect of an increase in the unemployment rate for marginal workers who were previously indifferent but now, due to (K1), prefer temporary to permanent contracts;

[17] Notice that, as implied by this example, (U2) has two equivalent variants: $\left|\frac{\theta p'(\theta)}{p(\theta)}\right| \le 0.5$, or alternatively, $\left|\frac{\theta q'(\theta)}{q(\theta)}\right| \ge 0.5$ for all $\theta \in (0,\infty)$.

[18] The value of 0.5 is also utilized in the quantitative part of this study; see the discussion contained in Section 4.

[19] Only an informal discussion is presented here; Appendix A.5 is devoted to provide technical details, including a formal statement (Proposition 3) and its proof.

(K3) A *negative* effect of an increase in the aggregate unemployment rate, which is induced by the increase in the unemployment rate for marginal workers in (K2).

Note that (U1) is enough to make (K2) overwhelmed by (K3), leading to a conclusion that a rise in κ drives the labor market tightness down.

Second, the response of the labor market tightness to an extension of the maximum duration of temporary contracts can be decomposed into four components:[20]

(L1) A *positive* effect of an increase in the surplus of forming a temporary match;

(L2) A *positive* effect of an increase in the unemployment rate for marginal workers who were previously indifferent but now, due to (L1), prefer temporary to permanent contracts;

(L3) A *negative* effect of a decrease in the unemployment rate for existing temporary workers;

(L4) An *ambiguous* effect of a change in the aggregate unemployment rate, which is jointly caused by the increase in the unemployment rate for marginal workers in (L2) and the decrease in the unemployment rate for existing temporary workers in (L3).

As witnessed in (L1)–(L4), one cannot sign the effect of a longer Λ in general. Accordingly, in Appendix A.5, I propose two sufficient conditions, one making (L1) outweigh (L3) and the other ensuring the positivity of (L4), to establish the overall positive effect of a longer Λ on the labor market tightness.

Welfare properties The last question addressed in this section is whether the stationary equilibrium in the decentralized economy can achieve the social planner's allocation.[21] The objective of the social planner is to choose the labor market tightness $\theta \in (0, \infty)$ and the marginal worker type $z_r \in [\underline{z}, \overline{z}]$ to maximize aggregate output (including home production) net of the firing costs (incurred by permanent contracts) and the vacancy costs subject to constraints associated with labor market configurations (such as the search frictions and the fixed duration of temporary contracts), namely,

$$\begin{aligned} \max_{\theta,\, z_r} & \int_{\underline{z}}^{z_r} \left[\{\ell(z) - u(z,\theta)\} yz + u(z,\theta)bz - \theta u(z,\theta)c \right] \mathrm{d}z \\ & + \int_{z_r}^{\overline{z}} \left[\{\ell(z) - u(z,\theta)\} (yz - \delta\kappa) + u(z,\theta)bz - \theta u(z,\theta)c \right] \mathrm{d}z \end{aligned} \tag{21}$$

subject to, recalling that $u_t(\theta) = \frac{\delta}{(1-\mathrm{e}^{-\delta\Lambda})p(\theta)+\delta}$ and $u_p(\theta) = \frac{\delta}{p(\theta)+\delta}$,

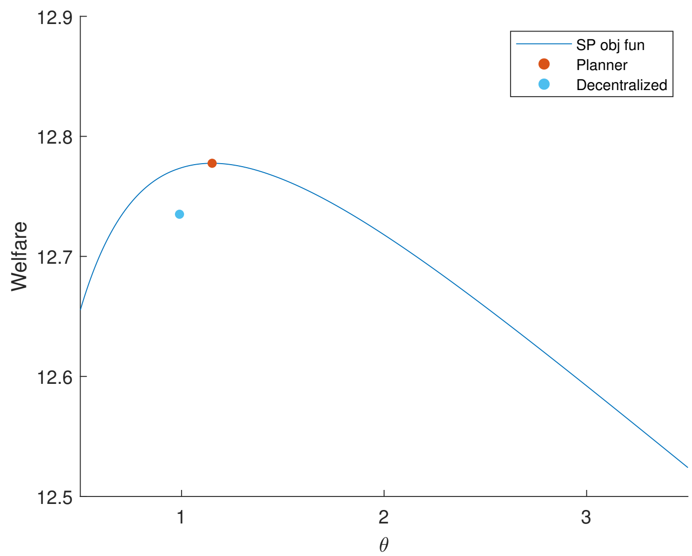
$$u(z,\theta) = \begin{cases} u_t(\theta)\ell(z) & \text{if } z < z_r, \\ u_p(\theta)\ell(z) & \text{if } z \geq z_r. \end{cases}$$

Let θ^P and z_r^P denote the labor market tightness and the marginal worker type, respectively, chosen by the planner, as opposed to θ^* and z_r^*, the corresponding objects in the decentralized economy. As apparently observed in (21), the planner has no interest in how to divide the surplus of a match between a worker and a firm, implying that both θ^P and z_r^P are independent of the exogenous bargaining power parameter β. Accordingly, the question of efficiency boils down to whether there exists a value of $\beta \in (0, 1)$ that ensures

[20] Contrary to the previous case, $u(z;\theta)$ in (17) is directly affected by a change in Λ, making the analysis more complicated.

[21] For the sake of simplicity, time discounting is ignored (that is, $r = 0$) in the following discussion.

Figure 4: A graphical illustration of the social planner's objective function in the benchmark model



Notes: The figure is drawn using the estimated parameter values reported in Table 2, with the values of ρ, ϕ_s, and ϕ_g all set to zero. Although it has no significant economic meaning, it is instructive to observe that the level of welfare implementable by the planner amounts to 12.78, a number 0.33 percent greater than that of the decentralized economy.

$\theta^* = \theta^P$ and $z_r^* = z_r^P$. Appendix A.6 is devoted to unraveling this question, thereby establishing the following proposition (with some technical assumptions).

Proposition 2 (Inefficiency of the decentralized economy)**.** *Suppose that the social planner's objective function (21) is uniquely maximized at $(\theta, z_r) = (\theta^P, z_r^P)$. Then there is no value of $\beta \in (0,1)$ that simultaneously guarantees $\theta^* = \theta^P$ and $z_r^* = z_r^P$. In other words, the stationary equilibrium is not efficient.*

Proof. See Appendix A.6. □

Figure 4 provides a graphical illustration of Proposition 2, and the intuition behind the proposition is as follows. Since the planner's optimal choice (θ^P, z_r^P) uniquely maximizes the objective function (21), the stationary equilibrium is efficient if and only if $\theta^* = \theta^P$ and $z_r^* = z_r^P$. However, the value of β which is implied by $\theta^* = \theta^P$ cannot coincide with the value of β which is pinned down by $z_r^* = z_r^P$. This is simply because $\theta^* = \theta^P$ and $z_r^* = z_r^P$ are two distinct objectives which are incompatible with each other: the former is required for an efficient total supply of jobs, whereas the latter is necessary for an optimal contract type choice. In a word, there is a "fundamental tension," as in Davis (2001) and Ljungqvist and Sargent (2012, Chapter 28), which prevents the decentralized economy to be efficient.[22]

[22] An alternative argument for Proposition 2 can be made based on the Hosios (1990) condition. Specifically, as the benchmark model belongs to the family of undirected search models, the standard Hosios condition $\beta = 1 - \eta(\theta^P)$ (where $\eta(\theta) = \theta p'(\theta)/p(\theta)$ so that $1 - \eta(\theta) = -\theta q'(\theta)/q(\theta)$) can be regarded as a necessary condition for efficiency. However, in general, $\beta = 1 - \eta(\theta^P)$ conflicts with $\beta = 1 - c[q(\theta^P)S_r]^{-1}$ that is required for $z_r^* = z_r^P$ (for instance, when the matching function takes the form of Cobb-Douglas, $\eta(\theta)$ becomes constant whereas $q(\theta)$ still depends on θ), leading to a failure of efficiency. For detailed arguments, see Appendix A.6, in which how to attain the socially optimal equilibrium is also discussed.

3 The Extended Model

The benchmark model developed and explored in Section 2 describes well the landscape of the dualized labor market, including the considerable share of temporary jobs, and the strong link between low education and temporary employment. However, much the same as models previously studied in the literature, this simple framework lacks a channel for human capital accumulation on the job. As a result, the role of training as an accelerator for the temporary-to-permanent conversion cannot be investigated within the benchmark model, calling for an extension of the model to incorporate human capital accumulation on the job—a main objective of this section. I extend the benchmark model by introducing two types of human capital: "general" and "specific" (Becker, 1964).[23] The main ingredients for the extension are largely borrowed from Flinn et al. (2017), and the key distinctions between the following extended model and theirs are drawn in Section 3.1. Section 3.2 redefines the stationary equilibrium in the new environment to study its welfare properties.

3.1 Setup

Agents The measure of workers is normalized to 1, as in the benchmark model. Each worker is born with a certain level of general human capital $z_0 \in \mathbb{Z}$, which is drawn from the cumulative distribution function $L(\cdot)$. Over their lifetimes, workers can increase the level of general human capital through on-the-job training when employed. To distinguish the current level of general human capital from its initial (innate) level, I denote the former by $z \in \mathbb{Z}$, where $\mathbb{Z}$ is assumed to be an evenly-spaced discrete set of N points (spaced by $\iota_z > 0$), namely, $\mathbb{Z} = \{z_1, \ldots, z_N\}$. Notice that depreciation of human capital is not modeled, and thus, $z = z_j$ is always greater than or equal to $z_0 = z_i$ for a given worker.[24] All workers are subject to a "retirement" (death) shock,[25] which arrives at Poisson rate $\rho > 0$. Each retired worker is replaced with an unemployed entrant who possesses the same *initial* level of general human capital.

Search and matching Unemployed workers and vacant jobs are brought together by a matching function $M(\cdot, \cdot)$. A match between a worker and a job is formed if and only if the expected joint match surplus is nonnegative. The expected joint match surplus is determined by their decision on the type of contract and the type of training. As before, there are two types of contracts (permanent and temporary contracts) available to a worker-firm pair. For each contract type chosen, it can be also decided whether to invest in training or not. If they agree to invest in training, the type of training (either specific or general training, but not both) can be chosen as well. To sum up, the choice set for a worker-firm pair consists of six options,[26] among which the best one (i.e. an option that yields the maximum expected joint match surplus) is chosen by the pair. As in the benchmark model, wages are determined by Nash bargaining over the match surplus, with $\beta \in (0, 1)$ being the bargaining power of workers.

[23] General human capital is defined as a type of human capital that is equally productive in all jobs. Specific human capital, on the other hand, is defined as a type of human capital that is productive in a given job but not in the other jobs.

[24] In other words, $j \in \{i, i+1, \ldots, N-1, N\}$ for a given $i \in \{1, \ldots, N\}$. Note that I use "z_j" and "z_i" throughout the paper to represent a typical value of z and z_0, respectively.

[25] The value of retirement is simply set to zero.

[26] For notational convenience, let p and t denote permanent and temporary contracts, respectively. Furthermore, let *n, s,* and g denote *no training, specific training,* and *general training,* respectively. Then the choice set for a worker-firm pair can be concisely described as $\{(p, n), (p, s), (p, g), (t, n), (t, s), (t, g)\}$.

On-the-job training I assume that every worker starts a new job (which can be either permanent or temporary) with the same level of specific human capital, denoted by y_0. It is further assumed that the production function has the form of yz (the product of the specific human capital level and the general human capital level), meaning that the flow output of a match associated with a worker whose specific human capital level equals y_0 and general one equals z is given by $y_0 z$ when all amount of time of a given moment (which is normalized to unity) is devoted to production.

If a permanent or temporary contract which stipulates specific training of a worker with general human capital z starts, an amount of time $\tau_s(z)$ is allocated for specific training at every moment of time until the training is complete. The completion of training follows a Poisson process with arrival rate ϕ_s. If the training is complete, then the level of specific human capital increases to $y_1 > y_0$. Endogenous job separation after the training completion is not allowed, but a new wage is negotiated via Nash bargaining over the match surplus that has been changed due to the increase in y. I assume that, once the training is complete, no further training is available until the match is exogenously separated.[27] In case of general training, everything is the same as the specific case, except that $\tau_g(z)$ and $\phi_g(z)$ replace $\tau_s(z)$ and ϕ_s, respectively, and that the completion of general training leads to an increase in the general human capital level from z to $z' = z + \iota_z$. In what follows, I suppose that $\tau_g(z) = \phi_g(z) = 0$ for $z = z_N$ (with $\phi_g(z) = \phi_g$ for $z < z_N$) so that a worker with $z = z_N$ cannot "break through the ceiling" through general training.

Surplus As in the benchmark model, one can formulate value functions for workers and firms under the extended environment. Omitting a detailed discussion on it, I delve into surplus functions in what follows.

For a worker of type $z \in \mathbb{Z}$, the surplus of a permanent contract that does not stipulate any type of training is denoted by $S_{p,n}(z)$. Following a similar procedure described in Section 2.1 (see (8), in particular), one can obtain

$$(r + \rho + \delta)S_{p,n}(z) = y_0 z - (r + \rho)W_u(z) - \delta\kappa, \tag{22}$$

where $W_u(z)$ is the value of unemployment so that it satisfies the extended counterpart of (7), namely,

$$(r + \rho)W_u(z) = bz + p(\theta)\beta S(z), \tag{23}$$

in which $S(z)$ represents the expected joint surplus accruing from the match (to be formally defined later).

Let $S_{p,s}(z)$ denote the surplus of a permanent contract that stipulates specific training of a worker with general human capital z. Then $S_{p,s}(z)$ satisfies the following equation:

$$(r + \rho + \delta)S_{p,s}(z) = [1 - \tau_s(z)]\, y_0 z + \phi_s \left[S^{p,s}_{p,s}(z) - S_{p,s}(z)\right] - (r + \rho)W_u(z) - \delta\kappa, \tag{24}$$

where $S^{p,s}_{p,s}(z)$ stands for the surplus that can be enjoyed after the completion of specific training, and it is formally described as[28]

$$(r + \rho + \delta)S^{p,s}_{p,s}(z) = y_1 z - (r + \rho)W_u(z) - \delta\kappa. \tag{25}$$

[27]This assumption is consistent with the finding of previous studies (e.g. Flinn et al., 2017) that workers receive training, typically, during the early period of employment.

[28]It is worthwhile noting that the outside option of the worker does not change even after the completion of specific training (because the specific human capital accumulated on the job is supposed to be fully depreciated upon destruction of the match), as reflected in (25).

If the worker-firm pair decides to invest in specific human capital, an amount of time $1 - \tau_s(z)$ is allocated to production while the remainder $\tau_s(z)$ is devoted to specific training. The investment in specific human capital becomes successful at rate ϕ_s, in which case the surplus is changed to $S^{p,s}_{p,s}(z)$. The third term on the right-hand side of (24) is related to the outside option of the worker. The last term appears since the permanent contract is currently under consideration.

The surplus delivered by a combination of the permanent contract and the general training for a worker with $z < z_N$ is denoted by $S_{p,g}(z)$. A similar argument as the case of $S_{p,s}(z)$ allows me to write

$$(r+\rho+\delta)S_{p,g}(z) = [1-\tau_g(z)]\, y_0 z + \phi_g\left[S^{p,g}_{p,g}(z') - S_{p,g}(z) + W_u(z') - W_u(z)\right] - (r+\rho)W_u(z) - \delta\kappa, \quad (26)$$

where $S^{p,g}_{p,g}(z')$ corresponds to the surplus that is updated after the completion of general training, whose formal definition is implied by

$$(r+\rho+\delta)S^{p,g}_{p,g}(z') = y_0 z' - (r+\rho)W_u(z') - \delta\kappa. \quad (27)$$

If a type-z worker receives general training, only an amount of time $1 - \tau_g(z)$ is used for production. The general training for the worker is completed at rate ϕ_g, which results in an "upgrade" of general human capital from z to $z' = z + \iota_z$. The growth of general human capital induces an adjustment not only in the match surplus but also in the worker's outside option (because the general human capital accumulated through training will not be destroyed upon separation of the match), as indicated in the second term on the right-hand side of (26). The last two terms show up for the same reason as before.

In order to calculate $S_{t,n}(z)$, the surplus of a temporary contract that does not stipulate any type of training for a worker with z, one can refer to the discussion in Section 2.1 (especially, (9)) to arrive at

$$(r+\rho+\delta)S_{t,n}(z) = [1-\mathrm{e}^{-(r+\rho+\delta)\Lambda}]\left[y_0 z - (r+\rho)W_u(z)\right]. \quad (28)$$

Recall that the temporary job that has survived until Λ must be destroyed without any exception, which occurs with probability $\mathrm{e}^{-(\rho+\delta)\Lambda} \in (0,1)$. For later purposes, let $\sigma_n := 1 - \mathrm{e}^{-(r+\rho+\delta)\Lambda}$ denote the "suppression" coefficient needed to calculate the "effective" surplus under (t,n).

If a worker z and a firm on a temporary contract agree to invest in specific human capital, the expected surplus, denoted by $S_{t,s}(z)$, has to satisfy

$$\begin{aligned}(r+\rho+\delta)S_{t,s}(z) = {} & [1-\mathrm{e}^{-R_s\Lambda}]\left[(1-\tau_s(z))y_0 z - (r+\rho)W_u(z)\right] \\ & +\phi_s\left[\int_0^\Lambda R_s \mathrm{e}^{-R_s\eta} S^{t,s}_{t,s}(z,\eta)\,\mathrm{d}\eta - S_{t,s}(z)\right],\end{aligned} \quad (29)$$

where $R_s := r+\rho+\delta+\phi_s$ is the effective discount rate, and $S^{t,s}_{t,s}(z,\eta)$ stands for the surplus that can be achieved if the specific training is completed at the moment that the worker's job tenure reaches $\eta \in [0,\Lambda]$, which is implicitly defined as

$$(r+\rho+\delta)S^{t,s}_{t,s}(z,\eta) = [1-\mathrm{e}^{-(r+\rho+\delta)(\Lambda-\eta)}]\left[y_1 z - (r+\rho)W_u(z)\right]. \quad (30)$$

When a temporary worker with z receives specific training, an amount of time $\tau_s(z)$ is invested in it, and thus, the match produces a flow of output $(1-\tau_s(z))y_0 z$ per unit of time. The term $(r+\rho)W_u(z)$ shown in the first line on the right-hand side of (29) represents the worker's outside option as before, while the term $1-\mathrm{e}^{-R_s\Lambda}$ in the first line is included to take into account the fact that the temporary job "surviving" until Λ (which is realized with probability $\mathrm{e}^{-(\rho+\delta+\phi_s)\Lambda}$) is inevitably destroyed at Λ. The second line of (29) is the counterpart of $\phi_s[S_{p,s}^{p,s}(z)-S_{p,s}(z)]$ in (24), indicating a surplus change initiated by the completion of specific training. For future purposes, it is convenient to define $\sigma_s := 1-\mathrm{e}^{-R_s\Lambda}$, the suppression coefficient required to obtain the effective surplus under (t,s), and

$$\tilde{\sigma}_s := \int_0^\Lambda R_s \mathrm{e}^{-R_s\eta}\left[1-\mathrm{e}^{-(r+\rho+\delta)(\Lambda-\eta)}\right]\mathrm{d}\eta,$$

the expected suppression coefficient necessary to calculate the effective after-specific-training-completion surplus under the temporary contract.[29]

Let $S_{t,g}(z)$ denote the surplus of a temporary contract that stipulates general training of a worker with general human capital $z<z_N$. In a similar fashion as above, one can derive

$$\begin{aligned}(r+\rho+\delta)S_{t,g}(z) &= [1-\mathrm{e}^{-R_g\Lambda}]\left[(1-\tau_g(z))y_0 z-(r+\rho)W_u(z)\right]\\ &\quad+\phi_g\left[\int_0^\Lambda R_g\mathrm{e}^{-R_g\eta}S_{t,g}^{t,g}(z',\eta)\,\mathrm{d}\eta - S_{t,g}(z)\right]+\phi_g[1-\mathrm{e}^{-R_g\Lambda}]\left[W_u(z')-W_u(z)\right],\end{aligned}\tag{31}$$

where $R_g := r+\rho+\delta+\phi_g$ is the effective discount rate, and $S_{t,g}^{t,g}(z',\eta)$ represents the surplus which is recalculated if the general training is completed at the moment that the worker's tenure on the job reaches $\eta\in[0,\Lambda]$, namely,

$$(r+\rho+\delta)S_{t,g}^{t,g}(z',\eta) = [1-\mathrm{e}^{-(r+\rho+\delta)(\Lambda-\eta)}]\left[y_0 z'-(r+\rho)W_u(z')\right].\tag{32}$$

The first line of (31) has the same interpretation as that of (29), and the second line can be regarded as the counterpart of $\phi_g[S_{p,g}^{p,g}(z')-S_{p,g}(z)+W_u(z')-W_u(z)]$ in (26). Note again that the general human capital accumulated via training will not be shattered upon termination of the contract. Therefore, both the match surplus and the worker's outside option need to be reevaluated in response to the increase in general human capital, as reflected in the last two terms of (31). Under the current context, σ_g and $\tilde{\sigma}_g$ can be similarly defined (and interpreted) as in the case of (t,s), namely, $\sigma_g := 1-\mathrm{e}^{-R_g\Lambda}$, and

$$\tilde{\sigma}_g := \int_0^\Lambda R_g\mathrm{e}^{-R_g\eta}\left[1-\mathrm{e}^{-(r+\rho+\delta)(\Lambda-\eta)}\right]\mathrm{d}\eta.$$

Lastly, given a worker with general human capital z, the expected joint surplus accruing from the match, $S(z)$, is formally defined as

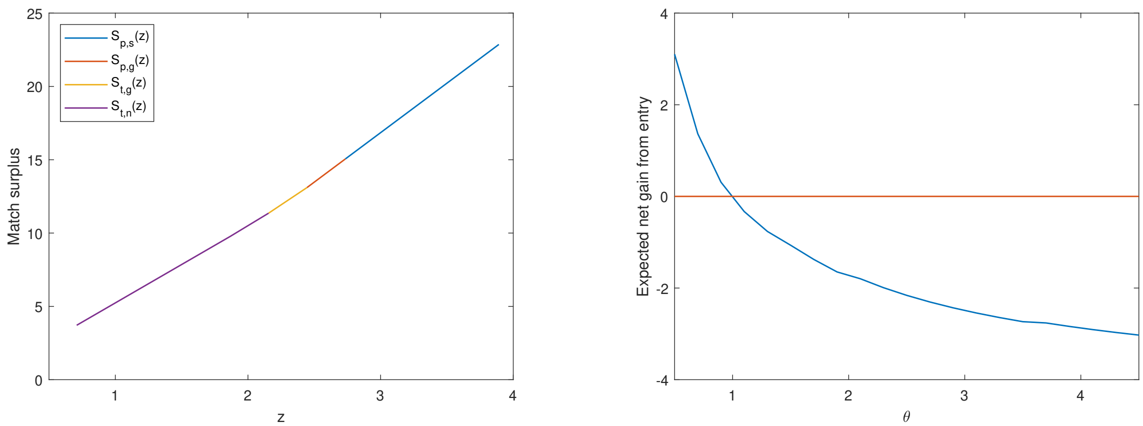
$$S(z) := \max\{S_{p,n}(z), S_{p,s}(z), S_{p,g}(z), S_{t,n}(z), S_{t,s}(z), S_{t,g}(z)\}.\tag{33}$$

See Figure 5.(a) for a numerical example of $S(\cdot)$.

[29] It will be useful later to observe that $\tilde{\sigma}_s$ can be equivalently expressed as $\sigma_s-\frac{R_s}{\phi_s}[\mathrm{e}^{-(r+\rho+\delta)\Lambda}-\mathrm{e}^{-R_s\Lambda}]=\sigma_s-\frac{R_s}{\phi_s}[\sigma_s-\sigma_n]$, and that it converges to 1 as Λ tends to infinity.

Figure 5: A graphical illustration of the extended model

(a) Surplus functions (b) The free-entry condition



Notes: Both figures are drawn based on the model specification described in Section 4.2 and the estimated parameter values reported in Table 2. The labor market tightness is normalized to one for estimation so that, given the estimated parameter values, $\theta = 1$ is the unique value that satisfies the free-entry condition (38).

Discussion on $S(z)$

(Q1) *Temporary contracts and on-the-job training.*

In the extended model, holding a temporary contract diminishes the incentive to invest in both types of human capital, which is consistent with the stylized fact discussed in Section 1. This implication from the model is intuitive: *ceteris paribus,* the period during which the temporary worker-firm match can gain the benefits from successful training is relatively short due to the predetermined duration of the temporary contract, thereby forcing them to hesitate on investing in any type of training. In what follows, I formally establish this implication for the case of specific human capital.[30]

In order to show that temporary employment results in a lower incentive to invest in specific training, I first study under what conditions a permanent worker-firm match invests in specific training, that is, $S_{p,n}(z) < S_{p,s}(z)$ for a given $z \in \mathbb{Z}$. For this purpose, I replace $(r+\rho)W_u(z)$ in (22) and (24) with the right-hand side of (23) to arrive at[31]

$$[r+\rho+\delta+p(\theta)\beta]\, S_{p,n}(z) = (y_0 - b)z - \delta\kappa,$$
$$[r+\rho+\delta+p(\theta)\beta]\, S_{p,s}(z) = (y_0 - b)z - \delta\kappa - \tau_s(z)y_0 z + \phi_s\left[S_{p,s}^{p,s}(z) - S_{p,s}(z)\right],$$

from which it follows that $S_{p,n}(z) < S_{p,s}(z)$ if and only if the costs of specific training are less than its

[30] In the case of specific human capital, Berton and Garibaldi (2012) and Cabrales et al. (2017) have arrived at the same conclusion, but their analysis has been carried out within a restricted environment (in which, for example, constant exogenous wages are assumed so that temporary and permanent workers receive the same wage). Establishing the same implication for general training is a little complicated because of the following reason. The benefits from general training result *partly* from an increase in the value of the worker's outside option, and the worker can enjoy such partial benefits *earlier* if he receives general training on a temporary contract. Thus, the amount of the partial benefits needs to be sufficiently small to arrive at the desired result. However, the key argument is the same, and thus, I omit the discussion on general training.

[31] To obtain the following expressions, permanent deviations are considered so that $S_{p,n}(z)$ or $S_{p,s}(z)$ is substituted for $S(z)$ when $(r+\rho)W_u(z)$ is replaced with $bz + p(\theta)\beta S(z)$.

expected benefits, namely,

$$\tau_s(z)y_0z < \phi_s\left[S^{p,s}_{p,s}(z) - S_{p,s}(z)\right].$$

Then one can use the fact that $R_s\left[S^{p,s}_{p,s}(z) - S_{p,s}(z)\right] = y_1z - [1-\tau_s(z)]\,y_0z$ to conclude that, under the permanent contract associated with a type-z worker, investment in specific human capital occurs if and only if

$$(r+\rho+\delta)\tau_s(z)y_0 < \phi_s(y_1-y_0). \tag{34}$$

Notice that, provided $\tau_s'(z) < 0$ for all $z \in \mathbb{Z}$,[32] the left-hand side of (34) is decreasing in z whereas the right-hand side is constant, implying that, if (34) holds for some $z \in \mathbb{Z}$, it is satisfied for all $z' > z$.

A condition under which a temporary worker-firm match invests in specific training can be derived in a similar way. More precisely, one can use (23) to rewrite (28) and (29) as follows:

$$[r+\rho+\delta+\sigma_n p(\theta)\beta]\,S_{t,n}(z) = \sigma_n(y_0-b)z,$$
$$[r+\rho+\delta+\sigma_n p(\theta)\beta]\,S_{t,s}(z) = \sigma_n(y_0-b)z - \sigma_n\tau_s(z)y_0z + \frac{\phi_s}{R_s}\tilde{\sigma}_sY_s(z),$$

where $Y_s(z) := y_1z - [1-\tau_s(z)]\,y_0z$ represents the change in flow output thanks to the completion of specific training. Therefore, $S_{t,n}(z) < S_{t,s}(z)$ if and only if $\sigma_n\tau_s(z)y_0z < \frac{\phi_s}{R_s}\tilde{\sigma}_sY_s(z)$, or equivalently,

$$\sigma_s(r+\rho+\delta)\tau_s(z)y_0 < \tilde{\sigma}_s\phi_s(y_1-y_0). \tag{35}$$

Again, the left-hand side of (35) is decreasing in z as long as $\tau_s'(z) < 0$ for all $z \in \mathbb{Z}$, suggesting that $S_{t,n}(\cdot)$ and $S_{t,s}(\cdot)$ cross at most once over $\mathbb{Z}$. Furthermore, since $\sigma_s > \tilde{\sigma}_s$, by comparing (35) with (34), one can easily reach the conclusion that temporary employment leads to a lower incentive for investment in specific human capital, as desired.

(Q2) *Cutoff human capital levels.*
The endogenous accumulation of general human capital makes it complicated to characterize the contract type choice and the training investment decision without specific parameter values. However, two simplifying assumptions (namely, no depreciation of general human capital and no availability of general training for those with $z = z_N$) substantially facilitate the analysis. Specifically, under these two assumptions, one can first study the problem faced by type-z_N workers and their potential employers, and then, based on the inspection result for them, one can tackle the problem associated with type z_{N-1}, and so on. Nevertheless, a complete analysis is still demanding, and thus, not pursued in this paper. Instead, guided by estimation results to be discussed below (see Section 4.3), I focus only on the case where (p,s), (p,g), (t,g), and (t,n) are selected by groups of workers with "high," "high-medium," "low-medium," and "low" levels of human capital, respectively.[33]

[32] Here it is implicitly assumed that $\tau_s(\cdot)$ is differentiable on its entire domain (which is supposed to be a superset of $[z_1, z_N]$). Consistent with this assumption, I restrict the function $\tau_s(\cdot)$ to have the form $\tau_s(z) = \overline{\tau}/z$ with $\overline{\tau}$ fixed as 1 when estimating the model; see Section 4.2.

[33] Accordingly, it is assumed that $S(z) \neq \max\{S_{p,n}(z), S_{t,s}(z)\}$ for any $z \in \mathbb{Z}$. Furthermore, it is also assumed that $|\mathbb{Z}| = N \geq 4$ in the following analysis.

Before proceeding further, it is convenient to introduce the following notation. Let $z_{p,g}^{p,s}$ be the largest $z_j \in \mathbb{Z} \setminus \{z_N\}$ such that $S(z_j) = S_{p,g}(z_j)$ and $S(z_j') = S_{p,s}(z_j')$ for all $z_j' \in \{z_{j+1}, \ldots, z_N\}$. Similarly, for a given $z_{p,g}^{p,s} \in \mathbb{Z}$, let $z_{t,g}^{p,g}$ be the largest $z_j \in \mathbb{Z} \setminus \{z_{p,g}^{p,s}, \ldots, z_N\}$ such that $S(z_j) = S_{t,g}(z_j)$ and $S(z_j') = S_{p,g}(z_j')$ for all $z_j' \in \{z_{j+1}, \ldots, z_{p,g}^{p,s}\}$. Lastly, for a given $(z_{p,g}^{p,s}, z_{t,g}^{p,g}) \in \mathbb{Z}^2$, let $z_{t,n}^{t,g}$ be the largest $z_j \in \mathbb{Z} \setminus \{z_{t,g}^{p,g}, \ldots, z_N\}$ such that $S(z_j) = S_{t,n}(z_j)$ and $S(z_j') = S_{t,g}(z_j')$ for all $z_j' \in \{z_{j+1}, \ldots, z_{t,g}^{p,g}\}$.[34] I am now ready to study how $z_{p,g}^{p,s}$, $z_{t,g}^{p,g}$, and $z_{t,n}^{t,g}$ are determined. Notice that, in order to facilitate the algebra, I specify the training cost functions $\tau_s(\cdot)$ and $\tau_g(\cdot)$ as $\tau_s(z) = \tau_g(z) = z^{-1}$, a functional form employed for the quantitative analysis later in the paper.

i. $S_{p,s}(z)$ *versus* $S_{p,g}(z)$.

Let $Y_g := y_0 z' - [1 - \tau_g(z)]\, y_0 z = y_0 \iota_z + y_0$ be the change in flow output owing to the completion of general training, and let $P(\theta) := \frac{p(\theta)\beta}{r+\delta+p(\theta)\beta}$ be the "effective job-finding rate" (for a given θ).

Suppose that workers with $z = z_N$ and their employers choose (p, s), that is, $S(z_N) = S_{p,s}(z_N)$. Then, assuming that all other options are dominated by (p, s) or (p, g) (to be confirmed later), one can find $z_{p,g}^{p,s}$ by successively (starting from z_{N-1} and potentially ending with z_1) checking whether $S_{p,s}(z) < S_{p,g}(z)$ is satisfied by a candidate $z = z_j$. In other words, $z_{p,g}^{p,s}$ is determined by the largest $z_j \in \mathbb{Z} \setminus \{z_N\}$ satisfying

$$\underbrace{r\left[\frac{\phi_s}{R_s} Y_s(z_j) - \frac{\phi_g}{R_g} Y_g\right]}_{\substack{\text{expected gains from } (p,s) \text{ relative to } (p,g) \\ \text{enjoyable at the current job}}} < \underbrace{\frac{\phi_g}{R_g}\delta\left[P(\theta)\left\{\frac{\phi_s}{R_s} y_1 + \frac{r+\delta}{R_s} y_0\right\} + (1 - P(\theta))b\right]\iota_z}_{\text{expected gains from } (p,g) \text{ enjoyable in the future}}, \tag{36}$$

where the left-hand side represents the expected gains from specific training that are enjoyable at the current job (relative to general training), whereas the right-hand side corresponds to the expected gains from general training enjoyable in the future (when the current match is destroyed).[35] Notice that the left-hand side is increasing in z_j while the right-hand side is independent of it, which verifies that $S(z_j') = S_{p,s}(z_j')$ for all $z_j' \in \{z_{p,g}^{p,s} + \iota_z, \ldots, z_N\}$.

ii. $S_{p,g}(z)$ *versus* $S_{t,g}(z)$.

The cutoff level $z_{t,g}^{p,g}$ is determined by the largest $z_j \in \mathbb{Z} \setminus \{z_{p,g}^{p,s}, \ldots, z_N\}$ satisfying

$$\begin{aligned} e^{-R_g \Lambda} R_g S_g(z) + \left[R_g + \left(1 + \frac{\phi_g}{r+\rho}\right) p(\theta)\beta\right] \phi_g \left(\sigma_g - \tilde{\sigma}_g\right) S_g^{p,g}(z') \\ < \left[R_g + \sigma_g \left(1 + \frac{\phi_g}{r+\rho}\right) p(\theta)\beta\right] \frac{R_g}{r+\rho+\delta} \delta\kappa, \end{aligned} \tag{37}$$

[34] For notational simplicity, both the dependence of $z_{t,g}^{p,g}$ on $z_{p,g}^{p,s}$ and the dependence of $z_{t,n}^{t,g}$ on $(z_{p,g}^{p,s}, z_{t,g}^{p,g})$ are suppressed. That is, I express $z_{t,g}^{p,g}(z_{p,g}^{p,s})$ and $z_{t,n}^{t,g}(z_{p,g}^{p,s}, z_{t,g}^{p,g}(z_{p,g}^{p,s}))$ simply as $z_{t,g}^{p,g}$ and $z_{t,n}^{t,g}$, respectively, unless any confusion arises. Meanwhile, it has to be remembered that $z_{p,g}^{p,s}$, $z_{t,g}^{p,g}$, and $z_{t,n}^{t,g}$ all depend on θ although notationally suppressed.

[35] Indeed, the right-hand side of (36) describes all possible cases that may occur to the worker when his current job is destroyed at rate δ after his general human capital level increases by ι_z at rate ϕ_g. When unemployed, the worker either does find a new job with "probability" $P(\theta)$, or does not with "probability" $1 - P(\theta)$. If he finds a new job, his specific human capital level could be either y_1 or y_0, depending on whether specific training is successfully completed with "probability" ϕ_s/R_s or not with "probability" $(r+\delta)/R_s$. If he fails to find a new job, the worker simply receives unemployment benefits b.

where $S_g^{p,g}(z') := (r+\rho+\delta)^{-1}\left[y_0 z' - (r+\rho)W_u(z')\right]$, and

$$S_g(z) := \left[y_0 - \left(1+\frac{\phi_g}{r+\rho}\right)b\right]z - y_0 + \phi_g\left[S_g^{p,g}(z') + W_u(z')\right].$$

The left-hand side of (37) stands for the relative costs of temporary contracts due to Λ, while the right-hand side corresponds to the relative costs of permanent contracts due to κ, when general training is available on both types of contracts. Therefore, if $\rho = \phi_g = 0$, (37) collapses to (14).

iii. $S_{t,g}(z)$ *versus* $S_{t,n}(z)$.
The procedure to characterize the cutoff level $z_{t,n}^{t,g}$ is less intuitive and more complicated. Thus, I relegate details to the online appendix to this paper (available upon request).

3.2 Stationary Equilibrium

Flow equations Analytically deriving the stationary transition equations in the current environment is a complicated task simply due to the endogenous dynamics of human capital accumulation. Consequently, for a thorough discussion, I refer readers to Appendix A.7 which includes an instructive example for the case of $N = 4$ (see Figure 11 therein). However, I introduce the related notation here for the following discussion.

For a given labor market tightness θ, let $\ell_i^j(\theta)$ denote the mass of workers whose initial general human capital level is $z_i \in \mathbb{Z}$ and current general human capital level is $z_j \geq z_i$.[36] Similarly, let $u_i^j(\theta)$ denote the mass of unemployed workers with $(z_0, z) = (z_i, z_j)$ with $i \leq j$. Meanwhile, I denote by $g_i^j(\theta)$ the mass of employed workers who were initially born with $z_0 = z_i$, and have completed the general training with the current employer so that his general human capital has increased from z_{j-1} to z_j. Lastly, I denote by $s_i^j(\theta)$ the mass of employed workers whose (z_0, z) equals (z_i, z_j) with $i \leq j$, and specific human capital is y_1 thanks to the completion of specific training on the current job.

Stationary equilibrium I slightly modify Definition 1 in Section 2.2 to define the stationary equilibrium of the extended model as follows (see Figure 5.(b) for a numerical example of the equilibrium).

Definition 2 (Equilibrium in the extended model)**.** A *stationary equilibrium* is a labor market tightness $\theta \in (0, \infty)$ satisfying the following free-entry condition:

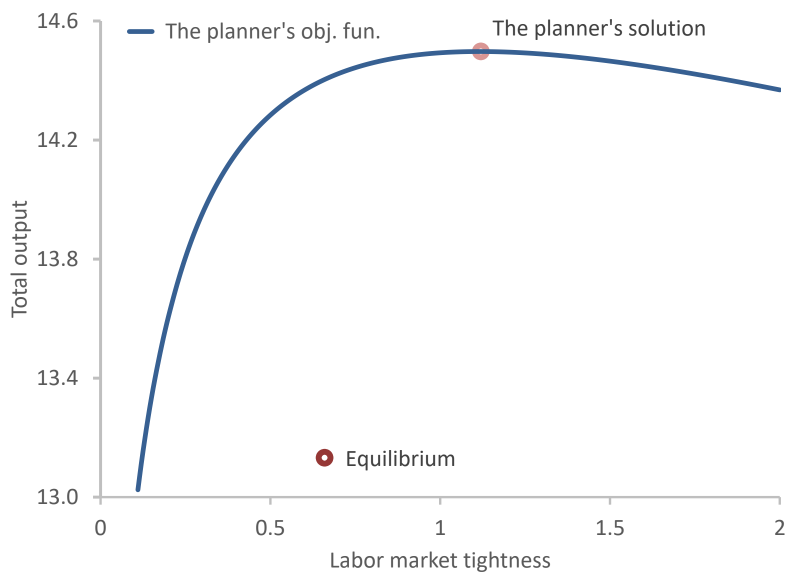
$$c = \sum_{z_j \in \mathbb{Z}} q(\theta)\frac{u(z_j;\theta)}{U(\theta)}(1-\beta)S(z_j;\theta), \tag{38}$$

where $S(z_j;\theta)$ is given by (33), $u(z_j;\theta) = \sum_{1\leq i\leq N} u_i^j(\theta)$, and $U(\theta) = \sum_{1\leq j\leq N} u(z_j;\theta)$.

Welfare properties The objective function of the social planner can be constructed in a similar manner to the benchmark case. The planner chooses the labor market tightness $\theta \in (0,\infty)$ and the set of options $\{(c(z_0), h(z_0))\}_{z_0\in\mathbb{Z}} \in \{(p,n),(p,s),(p,g),(t,n),(t,s),(t,g)\}^N$ to maximize aggregate output (including home production), net of the training costs, the firing costs (incurred by permanent contracts), and the vacancy costs, subject to constraints associated with labor market configurations (such as the search frictions and

[36] By definition, $\ell(z_i) = \sum_{j\geq i}\ell_i^j(\theta)$ for all $z_i \in \mathbb{Z}$, and $\sum_i\sum_{j\geq i}\ell_i^j(\theta) = \sum_i \ell(z_i) = 1$.

Figure 6: A graphical illustration of the social planner's objective function in the extended model



Notes: The figure is drawn based on the estimated parameters reported in Table 2, along with the model specification described in Section 4.2. The total output in equilibrium is 13.1, a number 9.4 percent below than the level achievable by the planner.

the fixed duration of temporary contracts).[37] Since a formal definition and analysis of the planner's objective function requires some more notation, I relegate further details to Appendix A.8, and discuss welfare implications in an informal way (see Figure 6 for a numerical illustration).

As claimed in Proposition 2, the stationary equilibrium of the benchmark economy (where training is not available) is not efficient. The extended model inherits this property so that the stationary equilibrium is not efficient as well. However, the availability of general training deteriorates the inefficiency because of positive externalities associated with general training (Acemoglu, 1997). Indeed, the benefits from general training are shared by the worker's future employers although they pay nothing for their benefits, leading to underinvestment in general training in the decentralized economy.[38] In Section 5, I quantitatively investigate this theoretical implication.

[37]To understand notation such as (p, s), see Footnote 26.

[38]On the other hand, specific training does not cause any inefficiency. To be specific, the investment decision is made after the match is formed, and accumulated specific human capital (if any) is completely depreciated when the match is destroyed. Thus, the labor market tightness does not affect the investment decision, and the benefits from specific training are not shared by any third party, both resulting in the socially optimal decision for specific training.

4 Estimation

4.1 Data

For the quantitative analysis, I utilize the 2002 to 2017 waves of the Korean Labor and Income Panel Study (KLIPS).[39] The KLIPS is an annually conducted panel survey on a sample of 5,000 Korean households and their members (aged 15 or over) designed to represent the nationwide population. In addition to standard individual characteristics (such as gender, age, and education), the data provide information on employment and labor market outcomes (such as wage, training participation—both at the extensive and at the intensive margins, and contract type). Accordingly, the data allow me not only to trace each individual's wage and training participation history, but also to observe contract type transitions occurring in the labor market (e.g. the transition rate from temporary to permanent contracts).

The descriptive statistics of the data are presented in Table 7 in Appendix B.1. The summary statistics confirm the stylized facts in the temporary employment literature: the low-educated are over-represented in temporary jobs, and temporary workers receive less on-the-job training than their permanent counterparts. I use a probit model to formally document the determinants of temporary employment and training incidence in the Korean labor market (Tables 8 and 9 in Appendix B.2). I also provide empirical evidence that training can accelerate the transition from temporary to permanent employment (Table 10 in Appendix B.2).

The results from the reduced-form analysis suggest that female workers are more over-represented in temporary jobs than their male counterparts, receiving much less on-the-job training. Thus, in the following quantitative analysis, I focus on female workers by estimating the model using a subsample consisting of female respondents aged 20 to 65 in 2010. The quantitative results regarding male and all workers are reported in Table 14 in Appendix B.3, and discussed in detail in the online appendix (available upon request).

4.2 Estimation Strategy and Identification

Model specification To specify the distribution of innate abilities $L(\cdot)$, I choose the gamma distribution, which is parameterized by two additional parameters μ_z and σ_z.[40] The functional form of the matching function is borrowed from the literature (e.g. Petrongolo and Pissarides, 2001) so that $M(U, V) = hU^{1-\eta}V^{\eta}$, where h is estimated whereas η is fixed as 0.5. I use $\tau_s(z) = \tau_g(z) = \overline{\tau}/z$, where $\overline{\tau}$ is fixed as 1 in the estimation stage. In Section 5.2, a decrease in $\overline{\tau}$ (i.e. a reduction in training costs) is allowed through subsidies financed by lump-sum taxes on employed workers, and its effects on the labor market are investigated.

The values of $(r, \rho, \beta, b, y_0, N)$ are set to be $(0.04, 0.04, 0.5, 3, 6, 12)$, respectively. I normalize a time period to be one year, and set the discount rate to $r = 0.04$ (corresponding to an annual discount factor of 0.96). I set the retirement shock to $\rho = 0.04$, which generates an average career length of $1/0.04 = 25$ years. Following the literature (e.g. Pissarides, 2009), I set the workers' bargaining power parameter to $\beta = 0.5$, and later examine the sensitivity of the quantitative results to variation of β (see Table 13 in Appendix B.3). The home production and the initial match productivity parameters are set to $b = 3$ and $y_0 = 6$, respectively. In the estimated model, this choice makes the ratio of the average unemployment benefit to the

[39] The year 2002 corresponds to the fifth wave of the KLIPS. I have opted to drop the first four waves because there were adjustments to the questionnaire between wave 4 (2001) and wave 5 (2002).

[40] Given a shape parameter $\mu_z > 0$ and a scale parameter $\sigma_z > 0$, the mean and variance of the gamma distribution are $\mu_z\sigma_z$ and $\mu_z\sigma_z^2$, respectively. For the purpose of estimation, the gamma distribution is truncated on the interval ranging from two standard deviations below its mean to one standard deviation above.

average wage amount to 0.52—a number very close to the unemployment benefit rate in Korea (50 percent of previous wages). Lastly, I have found it sufficient to use $\mathbb{Z}$ containing $N = 12$ points for the purpose of estimation, leading to a total of 10 parameters to estimate.

Estimation method To estimate the model, I use the method of simulated moments (MSM), a widely-used structural estimation technique (McFadden, 1989; Pakes and Pollard, 1989). Loosely speaking, the MSM finds the set of parameter values that minimizes the weighted difference between the actual data moments and the simulated data moments.[41] Formally, the MSM can be implemented by the following three steps. First, use the data to calculate a vector of moments $\widehat{m}_N$, where N is the number of observations in the sample. Second, choose a vector of parameter values, denoted by ψ, and generate S simulated careers from the model to construct $\widetilde{m}_S(\psi)$, the model counterpart to $\widehat{m}_N$. Lastly, iterate the second step until $\widehat{\psi}$, the vector of parameter values that brings $\widetilde{m}_S(\cdot)$ as close as possible to $\widehat{m}_N$, is found. Ultimately, the procedure of the MSM is summarized as follows:

$$\widehat{\psi} = \arg\min_{\psi \in \Psi} \left[\widehat{m}_N - \widetilde{m}_S(\psi)\right]' \widehat{W}_N \left[\widehat{m}_N - \widetilde{m}_S(\psi)\right],$$

where Ψ is the parameter space, and $\widehat{W}_N$ is a positive-definite weighting matrix.[42]

The practical use of the MSM estimator is often challenging mainly due to the large parameter space and the existence of many local minima of the objective function. Recent studies in the search and matching literature (e.g. Jarosch, 2015; Lise et al., 2016) overcome this difficulty by using the Markov chain Monte Carlo (MCMC) method developed by Chernozhukov and Hong (2003), and I rely on the same strategy. To be specific, I simulate a Markov chain with ergodic distribution, and compute the mean of the Markov chain to obtain a point estimate for the parameters. In addition, standard errors of the parameter estimates are obtained as the standard deviation of the Markov chain.

Identification A rigorous analysis of identification is beyond the scope of this paper, and thus, I present an identification argument in a heuristic way. To be specific, I discuss, among all moments to be used in the estimation, which one is (ones are) expected to be sensitive to a particular parameter. Then I support my choice of moments by reporting the elasticity of each moment with respect to each parameter (Table 11 in Appendix B.3).

The rate of improvement in specific or general human capital directly affects training investments. Thus, I utilize training participation rates of permanent and temporary workers (Moments S1 and S2 in Table 2) as key moments to identify ϕ_s and ϕ_g. In the model, the transition from temporary to permanent employment is solely explained by the increase in z through general training. Therefore, in order to discipline ϕ_g more precisely, I also include the rate of "transition" from temporary to temporary employment over the three-year interval (Moment S3), and the share of permanent workers who were, three years ago, temporary workers receiving on-the-job training (Moment S4). Higher firing costs are positively related to a higher incidence

[41] Examples of moments include the share of permanent or temporary workers who have received on-the-job training during the last year, the average job tenure of permanent or temporary workers, the mean wage of permanent or temporary workers, etc. See Table 2 for the set of moments used in estimation.

[42] I use the bootstrap to compute the variance of each element in $\widehat{m}_N$, and construct a diagonal matrix $\widehat{V}_N$ with diagonal entries equal to the estimated variances. Then I set $\widehat{W}_N = \widehat{V}_N^{-1}$ for the purpose of estimation.

of temporary employment. Accordingly, I inform κ by computing the share of temporary workers among all employees in the labor market (Moment S5).

Recall that the matching elasticity parameter is chosen to be $\eta = 0.5$, and hence, given the labor market tightness (to be targeted), the matching efficiency parameter h governs the job-finding rate of the unemployed. Thus, I discipline h by measuring the three-year unemployment-to-employment transition rate (Moment J1). How long a permanent job lasts depends on the job separation rate δ, while how long a temporary job lasts depends on both δ and Λ (the maximum duration of temporary contracts). Therefore, I target the average job tenure of permanent and temporary workers (Moments J2 and J3) to identify δ and Λ.

Wage levels are governed by general and specific human capital levels. Thus, I calculate the average log wages of permanent and temporary workers (Moments A1 and A2) to discipline μ_z, σ_z (the shape and scale parameters of the innate ability distribution), and y_1 (the specific human capital level after training). In my model, training occurs at the beginning of job spells. Therefore, comparing the average wage of all jobs with the average wage of new jobs can provide information on the amount of wage growth after training. Accordingly, I further inform (μ_z, σ_z, y_1) by including the ratio of the average wage of new temporary jobs to the average wage of new permanent jobs (Moment A3) as one of the targets.

At this stage, the only parameter not yet determined is the flow cost of vacancy posting c. It is clear from the free-entry condition (38) that the vacancy to unemployment ratio is affected by c. Thus, I identify c by targeting the labor market tightness (Moment T1). Note that my data set does not contain information on the tightness so that it is taken from macroeconomic data,[43] as in Lise et al. (2016).

As documented in Table 11 in Appendix B.3, all the employed moments move in the expected direction in response to a small change in parameters. For example, a small increase in κ (higher firing costs) or Λ (longer duration of temporary employment) leads to an increase in the value of Moment S5 (a higher share of temporary workers) or J3 (longer job tenure of temporary workers), as expected. Therefore, I am confident that the set of moments chosen is sufficient to identify all parameters of the model although I cannot pin down each parameter with a single moment. Lastly, I plot one-dimensional slices of the objective function of the MSM to illustrate how it behaves around the estimated parameters (see Figure 12 in Appendix B.3).

4.3 Estimation Results

Parameter estimates The estimation results are summarized in Table 2, where moments and parameters are partitioned (by dashed lines) into four groups according to the identification strategy although all parameters have been jointly estimated.

The first set of parameters is mainly related to the composition of the labor market (Moments S1 to S5 in Table 2). The estimated rates of increase in specific and general human capital are 0.687 and 0.296, respectively, which implies that it is relatively demanding to accumulate general human capital after labor market entry. This implication makes sense, considering comparative advantages of formal schools over firms in terms of providing general learning experiences (Flinn et al., 2017). Another parameter that significantly affects the labor market composition is κ, whose estimate is given by 8.322. This figure corresponds to 5.9 times the average monthly wage of permanent workers who do not receive training—a higher number than the previous estimate of 3.4 times (Lee, 2015).

[43]The National Statistical Office of Korea provides the relevant macroeconomic data, which is available at `http://kosis.kr/statHtml/statHtml.do?orgId=101&tblId=DT_1YL1101` (last accessed on November 1, 2019).

Table 2: Moments and estimates

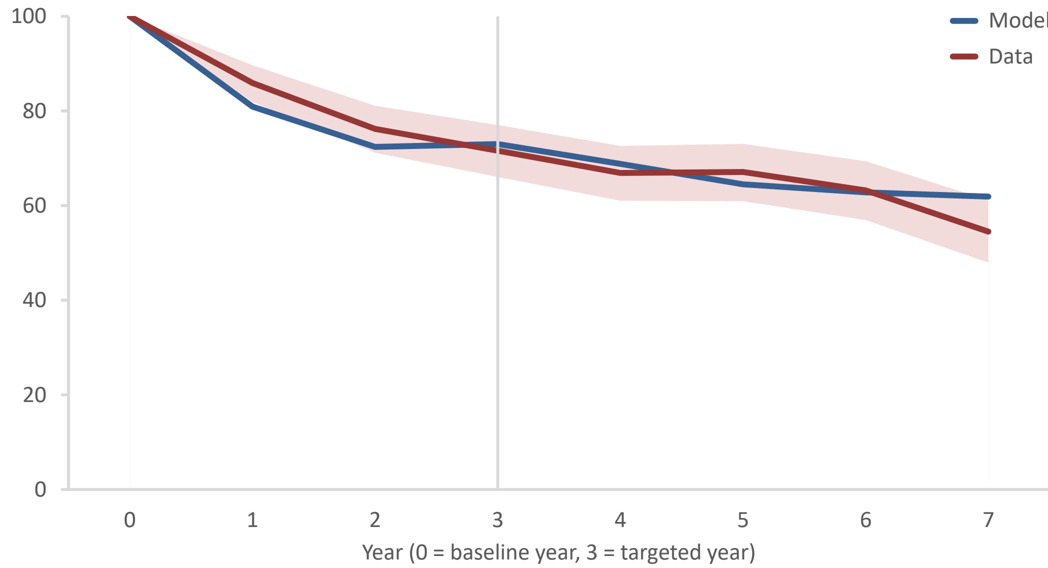
Moment	Model	Data	Parameter	Estimate
S1. share of p on training	0.184	0.174	rate of increase in y	$\phi_s = 0.687$
		(.018)		(.030)
S2. share of t on training	0.062	0.055	rate of increase in z	$\phi_g = 0.296$
		(.017)		(.013)
S3. share of t to t	0.730	0.716	firing costs for p	$\kappa = 8.322$
		(.018)		(.816)
S4. share of p who were t on training	0.001	0.002		
		(.033)		
S5. share of t in the labor force	0.288	0.290		
		(.002)		
J1. job-finding rate	0.789	0.815	matching efficiency	$h = 0.850$
		(.075)		(.060)
J2. job tenure of p	6.814	6.453	job separation rate	$\delta = 0.115$
		(.310)		(.007)
J3. job tenure of t	3.054	2.939	cap on t tenure	$\Lambda = 7.625$
		(.174)		(.394)
A1. avg log wage of p	2.757	2.739	avg innate abilities	$\mu_z = 6.421$
		(.026)		(.423)
A2. avg log wage of t	2.100	2.121	var innate abilities	$\sigma_z = 0.443$
		(.038)		(.024)
A3. avg log wage of new t relative to p	0.630	0.661	productivity after ϕ_s	$y_1 = 7.020$
		(.058)		(.089)
T1. tightness	0.661	0.581	vacancy cost	$c = 7.080$
		(.068)		(.269)

Notes: The twelve moments are employed to estimate ten parameters. Moments S4, S5, and J1 are calculated over the three-year interval. I use the bootstrap to compute the variance of each data moment, whose square root is reported in parentheses. All rate parameters are expressed at an annual frequency. Standard errors of the parameter estimates (obtained from the standard deviation of the converged MCMC chain) are reported in parentheses.

Match formation and destruction are governed by h, δ, and Λ in the model. The estimate of matching efficiency is 0.850, which, combined with the estimated labor market tightness 0.661, yields a monthly job-finding rate of $1-(1-h\sqrt{\theta})^{1/12} = 0.093$. Conversely, exogenous job separations occur once every $1/\delta = 8.7$ years for both permanent and temporary workers. However, the average job tenure of temporary workers is shorter than that of permanent workers due to the maximum duration of temporary employment. The estimated cap on the duration of temporary contracts reaches 7.6 years; this estimate is reasonable since the 95th percentile of the job tenure distribution of temporary workers is 7.3 years in the data.[44]

[44] Notice that temporary contracts renewed once or multiple times are regarded as a single employment relationship when

Figure 7: Temporary job trap—Temporary-to-temporary transition rates in the model and the data



Notes: The figure plots the share of temporary workers in $year \in \{0, 1, 2, 3, 4, 5, 6, 7\}$ among those who held temporary contracts in $year = 0$. While $year = 3$ was targeted in the estimation (Moment S4 in Table 2), the other years were not. The shaded area indicates a point-wise 90% confidence interval of the data moments.

Wages are essentially determined by general and specific human capital levels, and the former is fundamentally controlled by μ_z and σ_z while the latter by y_0 and y_1. With the specification of $\mathbb{Z}$ described in Section 4.2, the estimates of μ_z and σ_z imply that the average general human capital level of labor market entrants is 2.538, and that the variance is 0.635.[45] Furthermore, the estimated distance between adjacent general human capital levels is $\iota_z = 0.306$, corresponding to a roughly 8.4 (51.1) percent increase in general human capital after successful general training for workers with z equal to z_{N-1} (z_1, respectively). Meanwhile, the estimate of y_1 indicates that investment in specific human capital can increase it by approximately 17 percent (from $y_0 = 6$ to $y_1 = 7.020$).

Recall that the labor market tightness was targeted in the estimation (Moment T1). Thus, with the help of the free-entry condition (38), the vacancy cost is estimated to be 7.080.

Model fit The estimated model achieves a decent fit to the data. First of all, all the targeted moments are well matched, as presented in Table 2. Furthermore, the estimated model successfully reproduces some untargeted data moments as well. For instance, although only the three-year persistence rate in temporary employment was explicitly targeted in the estimation (Moment S4), the estimated model predicts well the persistence rates over shorter or longer time intervals, as illustrated in Figure 7.

On the other hand, the model's ability to replicate the wage moments is limited. While the average wage levels are well fitted by the model (see Moments A1 to A3 in Table 2), the variances are not: the variance of log wages for all workers in the simulated data is 0.145, approximately two-fifths of its actual counterpart. This is because there are few sources of wage dispersion in my model. For example, every worker starts a

calculating job tenure in the data.

[45] Appendix B.3 provides the estimated distribution of general human capital; see Figure 13 therein.

Table 3: Temporary job scar—Effects of starting a career with a temporary contract on current outcomes

	On-the-job training		*Log wage*	
	Model	Data	Model	Data
Constant	−0.566	−0.684***	2.785	3.304***
		(0.086)		(0.028)
First contract type				
Temporary contract	−0.096	−0.075	−0.242	−0.143***
		(0.096)		(0.031)
Current contract type				
Temporary contract	−0.684	−0.552***	−0.499	−0.529***
		(0.134)		(0.037)
Labor market experience				
(continuous, in years)	−0.007	−0.001	0.001	0.001
		(0.004)		(0.001)

Notes: The left panel of the table reports estimation results from a probit model whose dependent variable is *On-the-job training* (that takes the value of one if the individual has received training at least once during the previous year and zero otherwise). The right panel reports estimation results from a linear regression model whose dependent variable is $\log(wage)$. *Gender* (equal to one if female, and zero otherwise) was controlled in the analysis of the actual data, and the estimated coefficients are −0.216 and −0.553, respectively, in the left and right panels (with both significant at the 1 percent level). The number of observations in the actual data is 1,545, whereas 2,297 in the simulated data. Standard errors are reported in parentheses. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

new job with the same specific human capital, and thus, there are no differences in starting wages among workers with the same general human capital. Nevertheless, the model and the data are in general agreement on wage determinants, an issue I will revisit later in this section.

Effects of having a temporary contract in the first job In my model, on-the-job training is the only way for temporary workers to achieve permanent employment status. Consequently, my model is "wrong" in the sense that the effects of starting a career with a temporary contract on current labor market outcomes may be exaggerated. However, such extremeness allows my model to be a reference point. Indeed, one can estimate possible scarring effects of having a temporary contract in the first job using the actual data, and compare them with the model's predictions to evaluate how close or far the reality is to the "worst scenario."

Following this spirit, I measure the effects of the contract type in the first job on training participation and wages in the current job, using both real and simulated data. Specifically, I consider the following two auxiliary models:

$$\Pr(training = 1) = \Phi\left(\beta_0^t + \beta_1^t first + \beta_2^t current + \beta_3^t years\right), \tag{39}$$

$$\log(wage) = \beta_0^w + \beta_1^w first + \beta_2^w current + \beta_3^w years + \varepsilon, \tag{40}$$

where Pr denotes probability, Φ is the standard normal distribution function, $first$ is an indicator equal to one if the worker had a temporary contract in the first job and zero otherwise, $current$ is an indicator equal to one if the worker has a temporary contract in the current job and zero otherwise, $years$ represents the worker's years in the labor force, and ε is the error term.[46]

[46]The model (40) resembles the standard Mincer wage regression. To be specific, I replace the education and tenure variables with $first$ and $current$, respectively. Note that the worker's education level (job tenure, respectively) is highly correlated with

Table 4: Decomposition of the scarring effects on wages

	log($wage$)	log($wage$) components: log of …					
		z_0	z_τ	y_0	y_τ	$1-\tau$	$wage/Y$
Constant	2.785	1.112	0.000	1.792	0.116	−0.112	−0.123
First contract type							
Temporary contract	−0.242	−0.348	0.134	-	−0.017	−0.012	0.000
Current contract type							
Temporary contract	−0.499	−0.330	−0.134	-	−0.105	0.061	0.009
Labor market experience							
(continuous, in years)	0.001	−0.000	−0.000	-	0.000	0.001	−0.000

Notes: The first column of the table is the same as the third column of Table 3. In each row, the estimate in the first column (the coefficient from the regression using the log wage) must be equal to the sum of the other estimates in the same row; see (41) for the decomposition of log wages in the model. The number of simulated samples is 2,297.

The estimation results are presented in Table 3, in which the left and right panels correspond to (39) and (40), respectively. By inspecting the table, I derive the following three implications. First, in both panels, the coefficient estimates obtained from the simulated data are qualitatively well matched with those obtained from the actual data. In particular, the negative impact of starting a career as a temporary worker on current training and wages is successfully reproduced by the model. Second, those negative effects, however, are exaggerated in the simulated data. Again, in my model, on-the-job training is the only way for temporary workers to escape the "temporary job trap," and thus, the overestimation by the model is reasonable. Third, nevertheless, the difference between the actual scarring effects and the predicted ones is not huge. Indeed, the estimates of β_1^t and β_1^w based on the actual data amount to nearly four-fifths and three-fifths, respectively, of their simulated counterparts, which demonstrates the highly persistent response of training and wages to having a "bad" starting point in the Korean labor market.

The regression results reported in the right panel of Table 4 can be more deeply understood if one takes advantage of my structural model's ability to decompose the coefficients appearing in the right-hand side of (40). In order to see how each of $\{\beta_0^w, \beta_1^w, \beta_2^w, \beta_3^w\}$ can be decomposed into its quantifiable subfactors, let (slightly abusing notation) $y \in \{y_0, y_1\}$ denote a worker's current level of specific human capital. Then, note that y can be expressed as the multiplication of two terms: the initial specific human capital level of the worker (y_0), and the amount of specific human capital accumulated through specific training (denoted by y_τ). In a word, $y = y_0 y_\tau$. Furthermore, letting z_τ denote the amount of general human capital accumulated via general training allows me to write $z = z_0 z_\tau$. Now, recall that the amount of output produced by a firm hiring a worker with z and y is given by $zy(1-\tau)$, where τ denotes the costs of (either type of) training, or, in other words, the amount of time devoted to training.[47] Therefore, as in Flinn et al. (2017), one can arrive at the following equation:

$$\log(wage) = \log z_0 + \log z_\tau + \log y_0 + \log y_\tau + \log(1-\tau) + \log(wage/Y), \tag{41}$$

the worker's contract type in the first (current) job.

[47]Thus, τ can take the value of 0, $\tau_s(z)$, or $\tau_g(z)$, depending on the worker's training status.

where $Y := zy(1-\tau) = z_0 z_\tau y_0 y_\tau(1-\tau)$. It is evident from (41) that wages in the model consist of six elements: innate general human capital, additional general human capital acquired via training, initial specific human capital, additional specific human capital acquired via training, hours worked, and structural residuals.

The main benefit of such decomposition is that the effect of, for example, the first contract type on the current wage can be broken down into its effect on each component of the wage. Indeed, one can replace the left-hand side of (40) with each term of the right-hand side of (41) and run the regression (six times in total) to obtain the results reported in the last six columns of Table 4.[48] Now, inspecting the table helps interpret $\{\beta_0^w, \beta_1^w, \beta_2^w, \beta_3^w\}$ in terms of the six wage determinants in the model. Especially, the second row of the table reveals that the negative effect of the first contract type on the current wage (-0.242) is mostly associated with differences in innate general ability (-0.348). However, it is also disclosed that the contribution from differences in innate general ability is partly offset by the positive impact of general ability attained through training (0.134). Thus, one can conclude that, although on-the-job training functions as a "scar remover," its role is limited under the current labor market conditions.

As exemplified by the discussion on the ingredients of β_1^w, interpreting the coefficients of (40) based on the structural model is itself interesting and informative. However, the decomposition exercise will be more meaningful in the counterfactual analysis since it will help deeply understand changes in the scarring effects induced by a counterfactual scenario. Thus, what has been discussed here will be recollected in Section 5.

5 Counterfactual Analysis

As previously discussed in Section 3.2, the stationary equilibrium is not efficient. In other words, the decisions made by workers and firms on contract and training types in the decentralized economy do not coincide with the constrained planner's choice. The first goal of this section is thus to measure how far the decentralized economy is away from its potential in terms of output, and quantitatively attribute the output loss to distinct factors present in the model. The second objective is to explore possible sources of improvement in output through policy experiments. I investigate two policy alternatives: subsidizing permanent employment, and subsidizing on-the-job training. A balanced budget is imposed so that both policy options are financed via lump-sum taxes on employed workers. While permanent-job subsidies have been widely used and studied as a policy tool to address challenges from the rise of temporary employment, training subsidies have been largely overlooked in the relevant literature. The discussion in Section 5.2 thus sheds light on the current debate on labor market reforms.

5.1 Welfare Loss

Welfare loss One can quantify the welfare loss by assessing the equilibrium outcomes from the planner's perspective, and calculating the difference between the output level achieved by the planner and the equilibrium counterpart. Following this idea, I evaluate the planner's objective function (defined in Appendix A.8) at the decentralized equilibrium, thereby obtaining 13.1, the estimated output level of the current economy. This number corresponds to 90.6 percent of the constrained social optimum, suggesting that the welfare loss

[48]For each covariate, the coefficients from these six separate regressions must sum up to the coefficient from the regression using $\log(wage)$, an implication of (41).

of the decentralized economy amounts to 9.4 percent (see Figure 6 for a graphical illustration).[49]

Decomposition A welfare loss results from the discrepancy between the decentralized allocation and the planner's allocation. Specifically, different decisions on contract and training types are the factors that induce a welfare loss.[50] This observation implies that one can make the decentralized equilibrium approach the social optimum by correcting "wrong" decisions one by one. The socially inefficient decisions in the decentralized economy can be grouped into three categories: inefficient training decisions of permanent workers, inefficient training decisions of temporary workers, and inefficient contract-type choices. Therefore, I fix these types of wrong decisions one after another, documenting the amount of welfare improvement achieved by each step.[51]

The decomposition proceeds in three steps (see Figure 8.(a)). First, I rectify training decisions of permanent workers. In the estimated model, all permanent workers choose specific training. These decisions are inefficient from the point of view of the planner who can change only training decisions of permanent workers at this stage. All permanent workers (except for those with the highest general human capital level) receive general training in the planner's solution, which results in welfare gains of 38.7 percent of the output loss (corresponding to the bottom part of the center bar in Figure 8.(b)). The second step is to allow the planner to choose training types for both permanent and temporary workers. For permanent workers, the planner has the same solution as the first step (general training instead of specific one). For temporary workers, the planner prefers general training to no training. This replacement leads to output gains of 43.3 percent of the total loss (the middle part of the center bar in Figure 8.(b)). In the last step, the planner chooses both contract and training types. Thus, all the remaining loss (18.0 percent of the total loss; the top part of the center bar in Figure 8.(b)) is attributed to mismatched contract types (too many permanent jobs).

To sum up, the decomposition analysis reveals that the welfare loss is mostly caused by too little general training; the amount of the welfare loss attributable to mismatched contract types is relatively small. An implication from these results is that the welfare loss could be substantially diminished by a policy encouraging on-the-job training. I discuss this issue in detail in the following subsection.

5.2 Policy Experiments

Background Confronted with challenges from the rise of temporary employment, researchers and policymakers have been grappling with how to improve the labor market. The primary scheme suggested by researchers and implemented by politicians has been to reduce the use of temporary contracts.[52] However, as evidently implied by the fact that most developed countries are still struggling with the issues, policies purely aimed at reducing the gap between permanent and temporary contracts have not been effective. The Korean

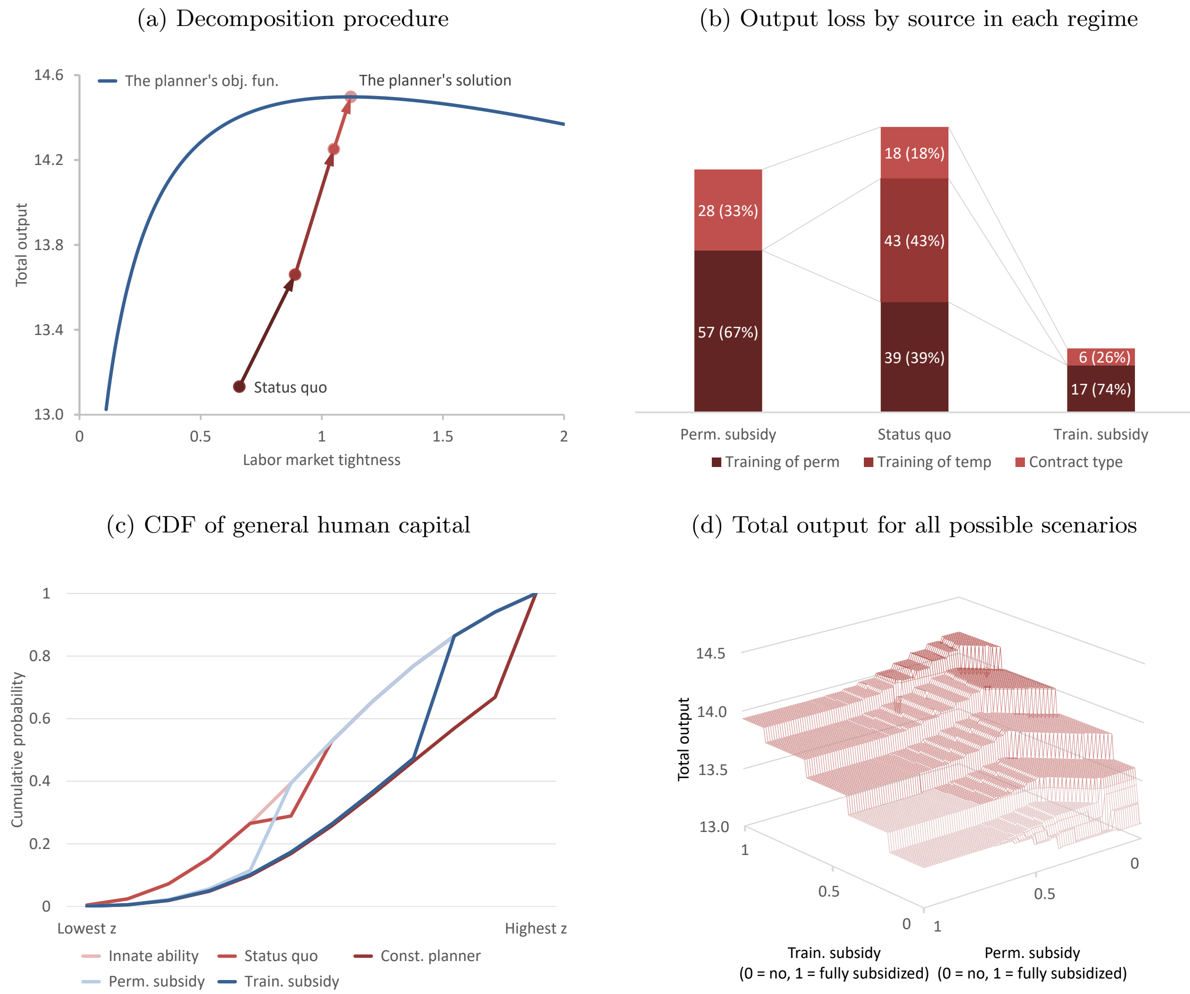
[49] Unfortunately, I have not been able to find studies providing comparable numbers in the literature, making this finding more meaningful despite its potential limitations.

[50] Recall that I set both β (the workers' bargaining power) and η (the elasticity of the matching function) to 0.5 in the estimation stage. Thus, there is no welfare loss attributable to externalities associated with the failure of the Hosios condition.

[51] The result of this decomposition procedure is obviously order-dependent. In other words, the decomposition result obtained from the "training-first-contract-later" rule could be different from that obtained from the "contract-first-training-later" rule. However, the robustness check indicates that the role of the decomposition order is limited from a qualitative perspective, although each possible order may yield different quantitative results. Therefore, instead of reporting all results from all possible orders, I focus only on the result obtained from the order that will be described below.

[52] One of the most recent studies on this type of policy alternative is Dolado et al. (2018). The authors propose a unified employment contract that is characterized by employment protection increasing with tenure, and explore the effects of replacing the currently-used two types of contracts with the proposed integrated one in the context of Spain, as earlier discussed by Cahuc (2012) for the French labor market.

Figure 8: Counterfactual analysis



Notes: (a) The figure is the same as Figure 6, but the equilibrium point is now named as "*Status quo*." The arrow starting from the *status quo* represents the first step where the planner is allowed to change training decisions of permanent workers. The second arrow departing from the destination of the first one corresponds to the second step where the planner chooses training types for all workers, regarding contract types as given. The third arrow stands for the last step where the planner solves his problem by choosing both contract and training types for all workers, thereby achieving the constrained social optimum denoted by the lightest-red dot. (b) The center bar represents the *status quo*; the left and right bars stands for counterfactual regimes where permanent employment and training, respectively, are subsidized at the optimal rates. For each bar, the dark-red (plain-red) segment corresponds to the output loss due to inefficient training decisions of permanent (temporary) workers, while the light-red part represents the output loss caused by inefficient contract-type choices. The amount of output loss in equilibrium is normalized to 100 (hence, the numbers on the center bar sum to 100), and the amount of output loss for other regimes is expressed in a relative manner (thus, the sum of numbers on the left or right bar stands for the remaining output loss under the corresponding regime). The numbers in parentheses report the relative contribution of each source in a given scenario.

and Spanish governments, for instance, attempted to reform the labor market in 2015 and 2010, respectively, by adopting more flexible employment protection regulations for permanent jobs, and enhancing job security for temporary employment, but failed to reach a political consensus. Both governments encountered strong objections from employers and both permanent and temporary employees (due to expected heterogeneous effects of the planned policy change), and the proposed legislation for the reform had to be withdrawn. Meanwhile, in Italy, labor market reforms were enforced by the authorities in 2001 despite fierce political resistance, which unfortunately caused unintended (and thus, undesirable) consequences, as comprehensively documented by Daruich et al. (2017). All these experiences not only demonstrate the difficulty of fixing the dysfunctional labor market, but also suggest a reason why it is necessary to consider and investigate a policy scheme that is beyond merely shrinking the gap and pursuing a single employment contract.

Policy options Motivated by the above background, I consider two policy options: subsidizing permanent employment, and subsidizing on-the-job training. The main objective of policy experiments is to investigate the effectiveness of training subsidies in terms of welfare improvement, relative to permanent-job subsidies. I impose a balanced budget condition, and thus, both policies are financed by lump-sum taxes on employed workers. The amount of subsidies for permanent employment is expressed in terms of firing costs. Thus, the amount of subsidies for each permanent job is bounded above by the estimated firing costs. Likewise, the amount of subsidies for training is expressed in terms of training costs. Recall that in the estimation, the training cost function was specified as $\tau_s(z) = \tau_g(z) = \overline{\tau}/z$, with $\overline{\tau}$ fixed as 1. I allow the planner choose $\overline{\tau}$, setting its lower and upper bounds to 0 and 1, respectively.

Under a balanced-budget constraint, the optimal permanent-job subsidy level is given by 0.68, meaning that the total output is maximized when each permanent job is subsidized by the amount of 68 percent of the estimated firing costs. In the case of training subsidies, the optimal subsidy level is determined by 0.76, meaning that the total output is maximized at $\overline{\tau} = 0.24$.[53] In what follows, I discuss the counterfactual effects of policy changes, assuming these two optimal cases.

Experiment results The experiment results are summarized in Table 5, in which columns (2) and (3) correspond to permanent-job and training subsidies, respectively. From the table, I draw out three implications. First, in terms of improvement in output, subsidizing training is more effective than subsidizing permanent employment. Training subsidies eliminate nearly 80 percent of the output loss, but permanent-job subsidies remove only 15 percent of the loss. Second, subsidizing permanent employment removes all temporary jobs in the economy, thereby inducing workers and firms to invest more in training. Its effect, however, is limited because most of workers receive specific training, rather than general one (see Figure 8.(c)). Third, the labor market under the training subsidy scenario emulates the constrained planner's economy well. Besides total output, other moment values related to the labor market composition and mobility are close to those of the efficient benchmark. This is mainly due to general training which is substantially encouraged by subsidies.

[53]For the amount of required taxes in both cases, see Table 5.

Table 5: A summary of policy experiments

	(1) *Status quo*	(2) Perm. subsidy	(3) Train. subsidy	(4) Const. planner
Welfare				
Output	100.00	101.55	108.07	110.40
Taxes	-	4.95	11.52	-
Labor market composition				
Moment T1	0.661	0.698	0.766	1.116
Moment S1	0.184	0.209	0.184	0.184
Moment S2	0.062	-	0.480	0.480
Moment S3				
− over 3 yrs	0.730	-	0.694	0.719
− over 5 yrs	0.645	-	0.622	0.690
− over 7 yrs	0.619	-	0.591	0.668
Moment S4				
− over 3 yrs	0.001	-	0.002	0.001
− over 5 yrs	0.001	-	0.014	0.016
− over 7 yrs	0.003	-	0.026	0.050
Moment S5	0.288	0.000	0.498	0.695
Wage				
Moment A2 over A1	0.762	-	0.833	0.759

Notes: The output level of the decentralized equilibrium has been normalized to 100 for easy comparison. The amount of taxes is also expressed in a relative manner. Although Moments S3 and S4 were previously defined and calculated over the three-year interval, the moment values calculated over the five- and seven-year intervals are also reported. Since there are no temporary workers when permanent jobs are optimally subsidized (the value of Moment S5 in column (2) is zero), the moments related to temporary workers have no values in column (2).

Table 6: Scarring effects on training in counterfactual scenarios

	(1) *Status quo*	(2) Perm. subsidy	(3) Train. subsidy	(4) Const. planner
Constant	−0.566	−0.498	−0.730	−0.686
First contract type				
Temporary contract	−0.096	-	−0.044	−0.080
Current contract type				
Temporary contract	−0.684	-	0.986	1.027
Labor market experience				
(continuous, in years)	−0.007	−0.007	−0.002	−0.002

Notes: Column (1) in the table is the same as the first column in Table 3. Since there are no temporary workers when permanent jobs are optimally subsidized, the coefficients of *First contract type* and *Current contract type* are not reported in column (2).

6 Conclusion

Despite the empirical evidence showing that the over-representation of the low-educated in temporary jobs, the low training incidence among temporary workers, and the role of training as a springboard to permanent employment, there has been little effort to understand the existence and persistence of temporary employment through the lens of human capital. I have developed and investigated a unified structural framework that can endogenously reproduce the stylized facts, thereby revealing the economic mechanisms behind them and studying their welfare implications.

In my model, workers with low general human capital choose temporary contracts because firing costs of permanent contracts are too burdensome, compared with the costs associated with frequent unemployment. Temporary workers, meanwhile, receive less training at work due to the short investment horizon, in addition to the human capital investment technology disfavoring those with low human capital. Thus, in equilibrium, low-ability workers are typically trapped in temporary jobs, as documented in the data. The decentralized equilibrium, however, turns out to be inefficient from the social point of view mainly because the social values of temporary employment and general training are not fully internalized in the decentralized economy.

The quantitative analysis, within the context of the Korean labor market, demonstrates the existence of both substantial scarring effects (of having a temporary contract in the first job) at the individual level, and sizable output losses at the economy-wide level. The output losses are largely attributed to suboptimal training decisions, rather than inefficient contract-type choices. More crucially, it is argued that temporary employment is not harmful by itself, but it becomes problematic when accompanied by the lack of training. The counterfactual experiments augment this argument by suggesting the following policy recommendation: activate the human capital accumulation channel inoperative in the presence of temporary contracts.

The structural framework developed in this paper can be a stepping stone to further research on un- or under-explored topics. First, although firms have been assumed to be identical throughout the paper, they may differ in many aspects. For instance, some firms may have short-term projects while others may have long-term ones, and accordingly, temporary contracts may be preferred by the firms with short-term projects (as in Cahuc et al., 2016). Then, it is interesting to ask, in the presence of temporary contracts, if there is assortative matching between workers and firms based on their characteristics (namely, human capital and job duration), and, if so, how strong and efficient the sorting is.

Meanwhile, it also has been assumed that workers enter the labor market with certain levels of education (general human capital) that are exogenously given. However, one may model the schooling decision (before entering the labor market), as in Flinn and Mullins (2015), to investigate how the existence of temporary employment affects, for example, the return to schooling, or the complementarity between education and training. A recent study by Charlot and Malherbet (2013) examined these issues within a simple framework (where workers choose the amount of education effort, but the contract type is randomly assigned), which can be combined with my model for comprehensive analysis.

Lastly, my counterfactual analysis is in fact comparative statics, meaning that I have been silent on the transition path between two steady states (one before the policy change, and one after the change). In order to more deeply understand how the labor market responds to policy changes, one may study the transition dynamics, as in Dolado et al. (2018). Constructing and tracking the dynamic trajectory, albeit challenging, would allow to explore unexamined issues, including how quickly the labor market adjusts to policy changes, and who are winners and losers along the path. I leave all these questions for future work.

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Appendices

A Additional Material - Theoretical Part

A.1 Formulating Bellman equations in the benchmark model

I focus only on the worker's problem since the firm's problem can be formulated in a similar way. Let $W_t(z, \lambda)$ be the value to worker $z \in \mathbb{Z}$ with tenure $\lambda \in [0, \Lambda]$ of holding a temporary contract.[54] Let $\mathrm{d}\lambda$ be a small interval of time. Standard dynamic programming arguments for continuous-time models (e.g. Cahuc et al., 2014, Appendix D) imply

$$W_u(z) = \frac{1}{1 + r\,\mathrm{d}\lambda} \left[bz\,\mathrm{d}\lambda + (1 - p(\theta)\,\mathrm{d}\lambda)\, W_u(z) + p(\theta)\,\mathrm{d}\lambda W(z)\right], \tag{A.1}$$

$$W_p(z) = \frac{1}{1 + r\,\mathrm{d}\lambda} \left[w_p(z)\,\mathrm{d}\lambda + (1 - \delta\,\mathrm{d}\lambda)\, W_p(z) + \delta\,\mathrm{d}\lambda W_u(z)\right], \tag{A.2}$$

$$W_t(z, \lambda) = \frac{1}{1 + r\,\mathrm{d}\lambda} \left[w_t(z)\,\mathrm{d}\lambda + (1 - \delta\,\mathrm{d}\lambda)\, W_t(z, \lambda + \mathrm{d}\lambda) + \delta\,\mathrm{d}\lambda W_u(z)\right], \tag{A.3}$$

for all $(z, \lambda) \in \mathbb{Z} \times [0, \Lambda)$. Rearranging terms in (A.1) and (A.2) directly yields (1) and (2), respectively. Manipulating (A.3) and then letting $\mathrm{d}\lambda \to 0$ results in

$$rW_t(z, \lambda) = w_t(z) + \delta\left[W_u(z) - W_t(z, \lambda)\right] + \frac{\partial W_t(z, \lambda)}{\partial \lambda}, \tag{A.4}$$

where $\frac{\partial W_t(z,\lambda)}{\partial \lambda} := \lim_{\mathrm{d}\lambda \to 0} \frac{W_t(z,\lambda + \mathrm{d}\lambda) - W_t(z,\lambda)}{\mathrm{d}\lambda}$. One can use the terminal condition $W_t(z, \Lambda) = W_u(z)$ to obtain the following solution to the differential equation (A.4):

$$rW_t(z, \lambda) = w_t(z) + \delta\left[W_u(z) - W_t(z, \lambda)\right] + \mathrm{e}^{-(r+\delta)(\Lambda - \lambda)}\left[rW_u(z) - w_t(z)\right],$$

for all $(z, \lambda) \in \mathbb{Z} \times [0, \Lambda]$.[55] Note that (3) is a special case obtained by setting $\lambda = 0$.

One can arrive at (3) in a different manner. Following the arguments in Cahuc et al. (2016), the value to worker z of starting a temporary job, $W_t(z)$, can be written as

$$W_t(z) = \underbrace{\int_0^{\Lambda} \left[\int_0^{\overline{\lambda}} \mathrm{e}^{-r\lambda} w_t(z)\,\mathrm{d}\lambda + \mathrm{e}^{-r\overline{\lambda}} W_u(z)\right] \delta \mathrm{e}^{-\delta\overline{\lambda}}\,\mathrm{d}\overline{\lambda}}_{\text{in case of a separation shock arriving before } \Lambda} + \underbrace{\left[\int_0^{\Lambda} \mathrm{e}^{-r\lambda} w_t(z)\,\mathrm{d}\lambda + \mathrm{e}^{-r\Lambda} W_u(z)\right] \mathrm{e}^{-\delta\Lambda}}_{\text{in case of a separation shock arriving after } \Lambda}. \tag{A.5}$$

Once the temporary job is started, a separation shock can arrive either before or after Λ. In the former case, the worker receives wage $w_t(z)$ until $\overline{\lambda} \leq \Lambda$, a random moment of time when the separation occurs so that the worker enters the pool of searchers with value $W_u(z)$. Recall that the separation occurrence follows the Poisson process whose density is $\delta \mathrm{e}^{-\delta\overline{\lambda}}$. In the latter case occurring with probability $\mathrm{e}^{-\delta\Lambda}$, the worker receives wage $w_t(z)$ until Λ, and then he goes back to unemployment. It is straightforward (albeit tedious) to verify that (A.5) is simplified to (3).

[54] Thus, $W_t(z, 0) = W_t(z)$ for all $z \in \mathbb{Z}$ by definition.

[55] The detailed procedure of deriving the solution is omitted for the sake of brevity, but one can easily verify the solution by substituting it into (A.4).

A.2 Surplus functions in the benchmark model

(10) and (11) constitute a system of two equations and two unknowns $S_p(z)$ and $S_t(z)$. Assuming that $z_r \in \mathbb{Z}$ as in the main text, one can solve the system for $S_p(z)$ and $S_t(z)$ as follows:

$$S_p(z) = \begin{cases} \dfrac{y-b}{r+\delta+[1-\mathrm{e}^{-(r+\delta)\Lambda}]p(\theta)\beta}z - \dfrac{\delta\kappa}{r+\delta} & \text{if } z < z_r, \\ \dfrac{y-b}{r+\delta+p(\theta)\beta}z - \dfrac{\delta\kappa}{r+\delta+p(\theta)\beta} & \text{if } z \geq z_r, \end{cases}$$

and

$$S_t(z) = \begin{cases} \dfrac{[1-\mathrm{e}^{-(r+\delta)\Lambda}](y-b)}{r+\delta+[1-\mathrm{e}^{-(r+\delta)\Lambda}]p(\theta)\beta}z & \text{if } z < z_r, \\ \dfrac{[1-\mathrm{e}^{-(r+\delta)\Lambda}](y-b)}{r+\delta+p(\theta)\beta}z + \dfrac{[1-\mathrm{e}^{-(r+\delta)\Lambda}]p(\theta)\beta}{(r+\delta)[r+\delta+p(\theta)\beta]}\delta\kappa & \text{if } z \geq z_r, \end{cases}$$

where it is worth mentioning that the slope of $S_p(z)$ is always steeper than that of $S_t(z)$. I can arrive at (12) by recalling that $S(z) = \max\{S_p(z), S_t(z)\}$.

A.3 Proof of Proposition 1

Part (a) Let $F(\theta)$ be the firm's expected net benefit from creating a vacancy as a function of the labor market tightness $\theta \in (0, \infty)$, namely,

$$F(\theta) := \int_{z\in\mathbb{Z}} q(\theta)\frac{u(z;\theta)}{U(\theta)}(1-\beta)S(z;\theta)\,\mathrm{d}z - c. \tag{A.6}$$

In order to prove the existence of $\theta^* \in (0, \infty)$ such that $F(\theta^*) = 0$, it is enough to show that

$$\underbrace{\left[\lim_{\theta\to 0} F(\theta)\right]}_{>0} \times \underbrace{\left[\lim_{\theta\to\infty} F(\theta)\right]}_{<0} < 0.$$

First, $\lim_{\theta\to 0} q(\theta) = \lim_{\theta\to 0} U(\theta) = 1$ and $\lim_{\theta\to 0} u(z;\theta) = \ell(z)$ jointly implies that

$$\lim_{\theta\to 0} F(\theta) = \int_{z\in\mathbb{Z}} \ell(z)(1-\beta)S(z;0)\,\mathrm{d}z - c,$$

which is strictly positive by (E2). Second, from the assumption $\lim_{\theta\to\infty} q(\theta) = 0$, it immediately follows that $\lim_{\theta\to\infty} F(\theta) = -c < 0$. Then the existence result is obtained by applying the intermediate value theorem with the fact that $F(\cdot)$ is continuous.

Part (b) I prove the uniqueness by showing that $F(\theta)$ is strictly monotone in θ.[56] Recalling (17), one can decompose $F(\theta)$ into three parts, $F(\theta) = F_t(\theta) + F_p(\theta) - c$, where

$$F_t(\theta) := \int_{\underline{z}}^{z_r(\theta)} \frac{q(\theta)u_t(\theta)}{U(\theta)} \ell(z)(1-\beta)S_t(z;\theta)\,\mathrm{d}z,$$
$$F_p(\theta) := \int_{z_r(\theta)}^{\overline{z}} \frac{q(\theta)u_p(\theta)}{U(\theta)} \ell(z)(1-\beta)S_p(z;\theta)\,\mathrm{d}z.$$

Let $\varepsilon[f(x)] := f'(x)/f(x)$ be the semi-elasticity of a continuously differentiable function $f(\cdot)$ at point x.[57] Using the Leibniz integral rule, I differentiate $F(\theta)$ with respect to θ, which yields

$$\begin{aligned} F'(\theta) = F_r'(\theta) + \varepsilon\left[\frac{q(\theta)u_t(\theta)}{U(\theta)}\right] F_t(\theta) + \varepsilon\left[\frac{q(\theta)u_p(\theta)}{U(\theta)}\right] F_p(\theta) \\ + \underbrace{\frac{q(\theta)u_t(\theta)}{U(\theta)} \int_{\underline{z}}^{z_r(\theta)} \ell(z)(1-\beta)\frac{\partial S_t(z;\theta)}{\partial\theta}\,\mathrm{d}z}_{<0} + \underbrace{\frac{q(\theta)u_p(\theta)}{U(\theta)} \int_{z_r(\theta)}^{\overline{z}} \ell(z)(1-\beta)\frac{\partial S_p(z;\theta)}{\partial\theta}\,\mathrm{d}z}_{<0}, \end{aligned}$$

where, letting $\ell_r(\theta) := \ell(z_r(\theta))$,

$$F_r'(\theta) := \frac{q(\theta)}{U(\theta)}\left[u_t(\theta) - u_p(\theta)\right] z_r'(\theta)\ell_r(\theta)(1-\beta)S_r. \tag{A.7}$$

Then, as $\frac{\partial S_t(z;\theta)}{\partial\theta} < 0$ and $\frac{\partial S_p(z;\theta)}{\partial\theta} < 0$, a sufficient condition for the negativity of $F'(\theta)$ is

$$F_r'(\theta) + \varepsilon\left[\frac{q(\theta)u_t(\theta)}{U(\theta)}\right] F_t(\theta) + \varepsilon\left[\frac{q(\theta)u_p(\theta)}{U(\theta)}\right] F_p(\theta) \le 0. \tag{A.8}$$

Noting that $U'(\theta)$ can be decomposed into two parts, $U'(\theta) = U_u'(\theta) + U_r'(\theta)$, where

$$U_u'(\theta) := u_t'(\theta)L_r(\theta) + u_p'(\theta)\left[1 - L_r(\theta)\right],$$
$$U_r'(\theta) := \left[u_t(\theta) - u_p(\theta)\right] z_r'(\theta)\ell_r(\theta),$$

one can rewrite (A.8) as

$$\underbrace{\underbrace{F_r'(\theta)}_{>0} - \underbrace{\frac{U_r'(\theta)}{U(\theta)}}_{>0}\underbrace{[F_t(\theta) + F_p(\theta)]}_{>0}}_{\le 0\ \forall\theta=\theta^*\text{ s.t. } F(\theta^*)=0\text{ by (U1)}} + \left[\underbrace{\underbrace{\varepsilon[q(\theta)]}_{<0} - \underbrace{\frac{U_u'(\theta)}{U(\theta)}}_{<0}}_{\le 0\text{ by (U2)}}\right]\underbrace{[F_t(\theta) + F_p(\theta)]}_{>0} + \underbrace{\sum_{i\in\{t,p\}} \varepsilon[u_i(\theta)]F_i(\theta)}_{<0} \le 0. \tag{A.9}$$

Therefore, (A.8) or (A.9) holds true, provided that the following two conditions are satisfied:

(Ua) $F_r'(\theta) - \frac{U_r'(\theta)}{U(\theta)}\left[F_t(\theta) + F_p(\theta)\right] \le 0$,

(Ub) $\varepsilon[q(\theta)] - \frac{U_u'(\theta)}{U(\theta)} \le 0$.

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[56] Technically speaking, I show that $F(\theta)$ is strictly decreasing at any $\theta = \theta^*$ such that $F(\theta^*) = 0$.

[57] It is worth noting that $\varepsilon\left[\frac{f(x)g(x)}{h(x)}\right] = \varepsilon[f(x)] + \varepsilon[g(x)] - \varepsilon[h(x)]$ for continuously differentiable functions $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$.

Now, to complete the proof, I show that (Ua) and (Ub) are implied by (U1) and (U2), respectively.

i. (U1) *implies* (Ua).
Since (A.7) can be simplified to $F_r'(\theta) = q(\theta)\frac{U_r'(\theta)}{U(\theta)}(1-\beta)S_r$, the left-hand side of (Ua) can be rewritten as

$$F_r'(\theta) - \frac{U_r'(\theta)}{U(\theta)}\left[F_t(\theta) + F_p(\theta)\right] = \frac{U_r'(\theta)}{U(\theta)}\left[q(\theta)(1-\beta)S_r - F_t(\theta) - F_p(\theta)\right],$$

which is negative as long as the following inequality holds:

$$q(\theta)(1-\beta)S_r \leq F_t(\theta) + F_p(\theta). \tag{A.10}$$

Suppose that there exists $\theta^* \in (0,\infty)$ such that $F(\theta^*) = 0$.[58] Then I need to check whether (A.10) holds at $\theta = \theta^*$. Recalling $F(\theta) = F_t(\theta) + F_p(\theta) - c$, one can see that

$$q(\theta^*)(1-\beta)S_r \leq (1-\beta)S_r \leq c = c + F(\theta^*) = F_t(\theta^*) + F_p(\theta^*),$$

where the first inequality holds since $q(\theta^*) \leq 1$, and the second inequality follows from (U1).

ii. (U2) *implies* (Ub).
It is straightforward to verify

$$\frac{u_p'(\theta)}{u_p(\theta)} = -\frac{p'(\theta)}{p(\theta)+\delta} < -\frac{(1-\mathrm{e}^{-\delta\Lambda})p'(\theta)}{(1-\mathrm{e}^{-\delta\Lambda})p(\theta)+\delta} = \frac{u_t'(\theta)}{u_t(\theta)},$$

which can be used to derive

$$\frac{u_p'(\theta)}{u_p(\theta)} < \frac{u_t'(\theta)L_r(\theta) + u_p'(\theta)\left[1-L_r(\theta)\right]}{u_t(\theta)L_r(\theta) + u_p(\theta)\left[1-L_r(\theta)\right]} = \frac{U_u'(\theta)}{U(\theta)}, \tag{A.11}$$

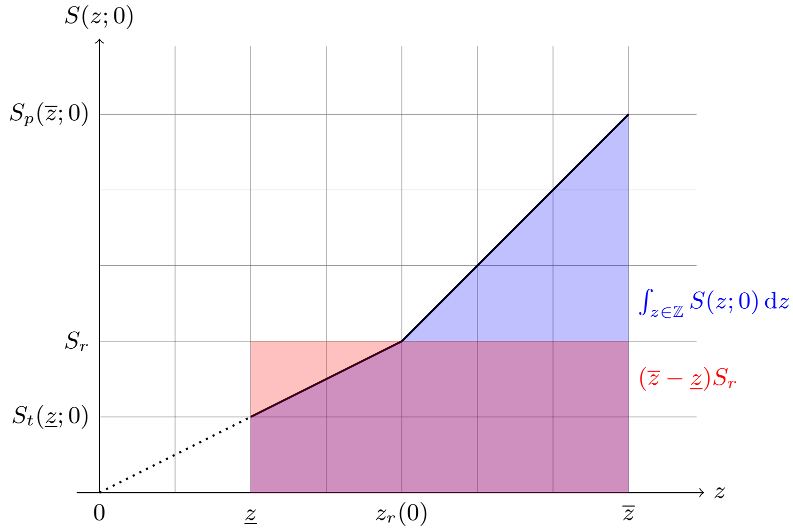
where the inequality is obtained by applying the mediant inequality. Then the desired result directly follows from

$$\varepsilon[q(\theta)] \leq -\varepsilon[p(\theta)] = -\frac{p'(\theta)}{p(\theta)} < -\frac{p'(\theta)}{p(\theta)+\delta} = \frac{u_p'(\theta)}{u_p(\theta)} < \frac{U_u'(\theta)}{U(\theta)},$$

where the first inequality is due to (U2), while the last inequality holds true by (A.11).

[58]Note that the existence of θ^* is guaranteed by (E1) and (E2).

Figure 9: A graphical illustration of the compatibility between (U1) and (E2)



Notes: The red area corresponds to the left-hand side of (A.12) while the blue area represents the right-hand side. It is clearly depicted in the figure that the former is smaller than the latter, with $z_r(0)$ being less than $(\underline{z}+\overline{z})/2$. The parameter values used to produce the figure are arbitrarily chosen only for the purpose of illustration.

A.4 Compatibility between (U1) and (E2)

Suppose that z is uniformly distributed over $[\underline{z},\overline{z}]$, with $(\underline{z}+\overline{z})/2 > z_r(0)$.[59] Then (20) is equivalent to

$$(\overline{z}-\underline{z})S_r < \int_{z\in\mathbb{Z}} S(z;0)\,\mathrm{d}z, \tag{A.12}$$

where the right-hand side can be rewritten as

$$\begin{aligned}\int_{z\in\mathbb{Z}} S(z;0)\,\mathrm{d}z &= \int_{\underline{z}}^{z_r(0)} S_t(z;0)\,\mathrm{d}z + \int_{z_r(0)}^{\overline{z}} S_p(z;0)\,\mathrm{d}z \\ &= \frac{1}{2}\left[z_r(0)-\underline{z}\right]\left[S_t(\underline{z};0)+S_r\right] + \frac{1}{2}\left[\overline{z}-z_r(0)\right]\left[S_r+S_p(\overline{z};0)\right].\end{aligned}$$

Therefore, (A.12) can be rearranged as $(\overline{z}-\underline{z})S_r < \left[z_r(0)-\underline{z}\right]S_t(\underline{z};0)+\left[\overline{z}-z_r(0)\right]S_p(\overline{z};0)$, or equivalently,

$$\left[z_r(0)-\underline{z}\right]\left[S_r-S_t(\underline{z};0)\right] < \left[\overline{z}-z_r(0)\right]\left[S_p(\overline{z};0)-S_r\right],$$

which holds true since $z_r(0)-\underline{z} < \overline{z}-z_r(0)$ by assumption, and $S_r-S_t(\underline{z};0) < S_p(\overline{z};0)-S_r$.[60] A graphical illustration is provided in Figure 9, in which the left-hand side of (A.12) is highlighted in red whereas the right-hand side is in blue.

[59] In other words, it is assumed that the mean (or median) worker type is greater than the marginal worker type determined when search frictions for unfilled jobs are absent.

[60] To see $S_r-S_t(\underline{z};0) < S_p(\overline{z};0)-S_r$, recall that the slope of $S_p(\cdot)$ is steeper than that of $S_t(\cdot)$, as illustrated in Figure 9.

A.5 Comparative statics for the benchmark model

In order to precisely state the comparative statics results for the benchmark model, I indicate whether an expression depends on κ or Λ by adding them to the notation as needed: e.g. $S_p(z;\theta,\kappa)$, $S_t(z;\theta,\Lambda)$, $F(\theta,\kappa)$, $F(\theta,\Lambda)$, etc. In addition, let $\varepsilon_{x_i}[f(x)] := [\partial f(x)/\partial x_i]/f(x)$ be the semi-elasticity of a multivariate continuously differentiable function $f:\mathbb{R}^n \to \mathbb{R}$ with respect to its i-th argument at point $x=(x_1,\ldots,x_i,\ldots,x_n)$.

Proposition 3 (Comparative statics).

(a) *Given the other model parameters satisfying* (E1) *and* (U2), *let* $\mathbb{K}$ *be the open interval containing all values of* κ *that (together with the other parameters) satisfy* (E2) *and* (U1). *Let* $\theta^*(\kappa_0)\in(0,\infty)$ *be the unique stationary equilibrium for some* $\kappa_0\in\mathbb{K}$ *so that* $F(\theta^*(\kappa_0),\kappa_0)=0$. *If* $F_\theta(\theta^*(\kappa_0),\kappa_0)\neq 0$, *then the slope of the level curve of* $F(\cdot,\cdot)$ *for the value* $F(\theta^*(\kappa_0),\kappa_0)$ *at the point* $(\theta^*(\kappa_0),\kappa_0)$, $\left.\frac{\mathrm{d}\theta^*(\kappa)}{\mathrm{d}\kappa}\right|_{\kappa=\kappa_0}$, *is strictly negative, namely,*

$$\left.\frac{\mathrm{d}\theta^*(\kappa)}{\mathrm{d}\kappa}\right|_{\kappa=\kappa_0} = -\frac{F_\kappa(\theta^*(\kappa_0),\kappa_0)}{F_\theta(\theta^*(\kappa_0),\kappa_0)} < 0.$$

(b) *Given the other model parameters satisfying* (E1) *and* (U2), *let* $\mathbb{L}$ *be the open interval containing all values of* Λ *that (together with the other parameters) satisfy* (E2) *and* (U1). *Let* $\theta^*(\Lambda_0)\in(0,\infty)$ *be the unique stationary equilibrium for some* $\Lambda_0\in\mathbb{L}$ *so that* $F(\theta^*(\Lambda_0),\Lambda_0)=0$. *If* $F_\theta(\theta^*(\Lambda_0),\Lambda_0)\neq 0$, *then the slope of the level curve of* $F(\cdot,\cdot)$ *for the value* $F(\theta^*(\Lambda_0),\Lambda_0)$ *at the point* $(\theta^*(\Lambda_0),\Lambda_0)$, $\left.\frac{\mathrm{d}\theta^*(\Lambda)}{\mathrm{d}\Lambda}\right|_{\Lambda=\Lambda_0}$, *is strictly positive, namely,*

$$\left.\frac{\mathrm{d}\theta^*(\Lambda)}{\mathrm{d}\Lambda}\right|_{\Lambda=\Lambda_0} = -\frac{F_\Lambda(\theta^*(\Lambda_0),\Lambda_0)}{F_\theta(\theta^*(\Lambda_0),\Lambda_0)} > 0,$$

provided that

(C1) $\varepsilon_\Lambda[S_t(z;\theta^*(\Lambda_0),\Lambda_0)] + \varepsilon_\Lambda[u_t(\theta^*(\Lambda_0),\Lambda_0)] \geq 0$,

(C2) $\varepsilon_\Lambda[U(\theta^*(\Lambda_0),\Lambda_0)] \leq 0$.

Before proving the above proposition, I provide concise interpretations of (C1) and (C2).[61] First, because $\partial S_t(z;\theta,\Lambda)/\partial\Lambda > 0$ for all $(z,\theta,\Lambda)\in\mathbb{Z}\times(0,\infty)\times\mathbb{L}$ (the longer is Λ, the higher is the surplus from a temporary match) and $\partial u_t(\theta,\Lambda)/\partial\Lambda < 0$ for all $(\theta,\Lambda)\in(0,\infty)\times\mathbb{L}$ (the longer is Λ, the lower is the unemployment rate for temporary workers), (C1) requires that $S_t(z;\theta,\Lambda)$ must be more elastic (with respect to Λ) than $u_t(\theta,\Lambda)$ at a given equilibrium $\theta^*(\Lambda_0)$.[62] Second, $\varepsilon_\Lambda[U(\theta,\Lambda)]$ can be decomposed into two parts, namely,

$$\varepsilon_\Lambda[U(\theta,\Lambda)] = \underbrace{[u_t(\theta,\Lambda)-u_p(\theta)]\frac{\partial z_r(\theta,\Lambda)}{\partial\Lambda}\ell_r(\theta,\Lambda)}_{>0} + \underbrace{\frac{\partial u_t(\theta,\Lambda)}{\partial\Lambda}L_r(\theta,\Lambda)}_{<0},$$

where the first part corresponds to an increase in the unemployment rate (caused by marginal workers who choose a temporary contract, instead of a permanent one, in response to a longer Λ), while the second part stands for a decrease in the unemployment rate (resulting from a reduced unemployment rate for existing temporary workers). Therefore, (C2) requires that, at a given equilibrium $\theta^*(\Lambda_0)$, the latter must outweigh the former in order for the net effect of a longer Λ on the aggregate unemployment rate to be negative.

[61] As will be clear in what follows, (C1) is introduced to ensure that (L1) is not overshadowed by (L3), while (C2) is imposed to guarantee the positivity of (L4).

[62] Note that $\varepsilon_\Lambda[S_t(z;\theta,\Lambda)]$ is independent of z, as seen in the proof below.

Proof. Since (E1)–(E2) and (U1)–(U2) are assumed, it immediately follows that $F_\theta(\theta^*(\kappa_0),\kappa_0) < 0$ and $F_\theta(\theta^*(\Lambda_0),\Lambda_0) < 0$ (see Appendix A.3). Therefore, by the implicit function theorem, it is enough to show that $F_\kappa(\theta^*(\kappa_0),\kappa_0) < 0$ and $F_\Lambda(\theta^*(\Lambda_0),\Lambda_0) > 0$ for parts (a) and (b), respectively.

Part (a) Recalling (A.6), one can differentiate $F(\theta,\kappa)$ with respect to κ, which yields

$$F_\kappa(\theta,\kappa) = \frac{\partial}{\partial\kappa}\left[\int_{\underline{z}}^{z_r(\theta,\kappa)} \frac{q(\theta)u_t(\theta)}{U(\theta,\kappa)}\ell(z)(1-\beta)S_t(z;\theta)\,\mathrm{d}z\right] + \frac{\partial}{\partial\kappa}\left[\int_{z_r(\theta,\kappa)}^{\overline{z}} \frac{q(\theta)u_p(\theta)}{U(\theta,\kappa)}\ell(z)(1-\beta)S_p(z;\theta,\kappa)\,\mathrm{d}z\right].$$

Using the fact that $\frac{\partial S_t(z;\theta)}{\partial\kappa} = 0$, I apply the Leibniz integral rule to obtain

$$\begin{aligned} F_\kappa(\theta,\kappa) = {}& \frac{q(\theta)}{U(\theta,\kappa)}\left[u_t(\theta)-u_p(\theta)\right]\frac{\partial z_r(\theta,\kappa)}{\partial\kappa}\ell_r(\theta,\kappa)(1-\beta)S_r(\kappa) + \varepsilon_\kappa\left[\frac{1}{U(\theta,\kappa)}\right]\left[F_t(\theta,\kappa)+F_p(\theta,\kappa)\right] \\ &+ \underbrace{\int_{z_r(\theta,\kappa)}^{\overline{z}} \frac{q(\theta)u_p(\theta)}{U(\theta,\kappa)}\ell(z)(1-\beta)\frac{\partial S_p(z;\theta,\kappa)}{\partial\kappa}\,\mathrm{d}z}_{=\,(\mathrm{K1})\,<\,0}, \end{aligned}$$

where the last term is less than zero as $\frac{\partial S_p(z;\theta,\kappa)}{\partial\kappa} < 0$. Consequently, a sufficient condition for $F_\kappa(\theta^*(\kappa_0),\kappa_0)$ to be negative is that

$$\underbrace{\frac{q(\theta)}{U(\theta,\kappa)}\left[u_t(\theta)-u_p(\theta)\right]\frac{\partial z_r(\theta,\kappa)}{\partial\kappa}\ell_r(\theta,\kappa)(1-\beta)S_r(\kappa)}_{=\,(\mathrm{K2})\,>\,0}\underbrace{-\,\varepsilon_\kappa\left[U(\theta,\kappa)\right]\left[F_t(\theta,\kappa)+F_p(\theta,\kappa)\right]}_{=\,(\mathrm{K3})\,<\,0} \le 0 \tag{A.13}$$

holds for $(\theta,\kappa) = (\theta^*(\kappa_0),\kappa_0)$. Note that $\frac{\partial U(\theta,\kappa)}{\partial\kappa} = \left[u_t(\theta)-u_p(\theta)\right]\frac{\partial z_r(\theta,\kappa)}{\partial\kappa}\ell_r(\theta,\kappa)$ can be used to derive

$$\varepsilon_\kappa\left[U(\theta,\kappa)\right] = \frac{1}{U(\theta,\kappa)}\left[u_t(\theta)-u_p(\theta)\right]\frac{\partial z_r(\theta,\kappa)}{\partial\kappa}\ell_r(\theta,\kappa),$$

which simplifies (A.13) to

$$q(\theta)(1-\beta)S_r(\kappa) - \left[F_t(\theta,\kappa)+F_p(\theta,\kappa)\right] \le 0. \tag{A.14}$$

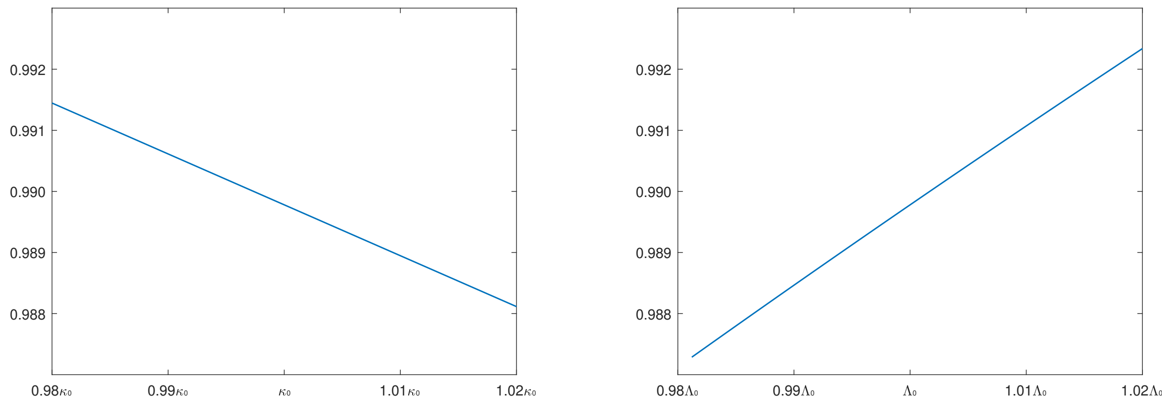
Then, since $F_t(\theta^*(\kappa_0),\kappa_0) + F_p(\theta^*(\kappa_0),\kappa_0) = c$, it is straightforward to verify that (U1) ensures (A.14) when $(\theta,\kappa) = (\theta^*(\kappa_0),\kappa_0)$, completing the proof.

Part (b) Noting that $\frac{\partial S_p(z;\theta)}{\partial\Lambda} = 0$, one can apply the Leibniz integral rule to obtain

$$\begin{aligned} F_\Lambda(\theta,\Lambda) = {}& \int_{\underline{z}}^{z_r(\theta,\Lambda)} \frac{q(\theta)u_t(\theta,\Lambda)}{U(\theta,\Lambda)}\ell(z)(1-\beta)\frac{\partial}{\partial\Lambda}S_t(z;\theta,\Lambda)\,\mathrm{d}z \\ &+\varepsilon_\Lambda\left[u_t(\theta,\Lambda)\right]F_t(\theta,\Lambda) + \varepsilon_\Lambda\left[\frac{1}{U(\theta,\Lambda)}\right]\left[F_t(\theta,\Lambda)+F_p(\theta,\Lambda)\right] \\ &+ \underbrace{\frac{q(\theta)}{U(\theta,\Lambda)}\left[u_t(\theta,\Lambda)-u_p(\theta)\right]\frac{\partial z_r(\theta,\Lambda)}{\partial\Lambda}\ell_r(\theta,\Lambda)(1-\beta)S_r(\Lambda),}_{=\,(\mathrm{L2})\,>\,0} \end{aligned}$$

Figure 10: An illustration of comparative statics for the benchmark model

(a) Labor market tightness as a function of κ (b) Labor market tightness as a function of Λ



Notes: (a) and (b) show how labor market tightness responds to small changes (up to $\pm 2\%$) in κ and Λ, respectively, in the benchmark case (where $\rho = \phi_s = \phi_g = 0$). The values of κ and Λ are initially set to $\kappa_0 = 6.087$ and $\Lambda_0 = 7.660$, respectively (the numbers reported in Table 2), yielding a unique stationary equilibrium $\theta^*(\kappa_0) = \theta^*(\Lambda_0) = 0.990$. The function $\theta^*(\kappa)$ is decreasing on the interval $(0.98\kappa_0, 1.02\kappa_0)$ in (a) whereas the function $\theta^*(\Lambda)$ is increasing on the interval $(0.98\Lambda_0, 1.02\Lambda_0)$ in (b), as established in Proposition 3.

where the last term is greater than zero as $\frac{\partial z_r(\theta,\Lambda)}{\partial \Lambda} > 0$. Now, it is useful to observe that $\varepsilon_\Lambda[S_t(z;\theta,\Lambda)]$ does not depend on z, as shown in

$$\varepsilon_\Lambda[S_t(z;\theta,\Lambda)] = \frac{(r+\delta)^2[\mathrm{e}^{(r+\delta)\Lambda} - 1]^{-1}}{r+\delta+[1-\mathrm{e}^{-(r+\delta)\Lambda}]p(\theta)\beta},$$

suggesting a sufficient condition for $F_\Lambda(\theta^*(\Lambda_0);\Lambda_0)$ to be positive:

$$\underbrace{\varepsilon_\Lambda[S_t(z;\theta,\Lambda)]F_t(\theta,\Lambda)}_{=(\mathrm{L1})>0} \underbrace{+\varepsilon_\Lambda\left[u_t(\theta,\Lambda)\right]F_t(\theta,\Lambda)}_{=(\mathrm{L3})<0} \underbrace{-\varepsilon_\Lambda\left[U(\theta,\Lambda)\right]\left[F_t(\theta,\Lambda)+F_p(\theta,\Lambda)\right]}_{=(\mathrm{L4}),\text{ whose sign is ambiguous}} \geq 0,$$

or equivalently,

$$\Big[\underbrace{\varepsilon_\Lambda\left[S_t(z;\theta,\Lambda)\right]+\varepsilon_\Lambda\left[u_t(\theta,\Lambda)\right]}_{\geq 0 \text{ at } (\theta,\Lambda)=(\theta^*(\Lambda_0),\Lambda_0) \text{ by (C1)}}\Big]F_t(\theta,\Lambda) \geq \underbrace{\varepsilon_\Lambda\left[U(\theta,\Lambda)\right]}_{\leq 0 \text{ by (C2)}}\left[F_t(\theta,\Lambda)+F_p(\theta,\Lambda)\right] \tag{A.15}$$

holds for $(\theta,\Lambda) = (\theta^*(\Lambda_0),\Lambda_0)$. Note that, when evaluated at $(\theta,\Lambda) = (\theta^*(\Lambda_0),\Lambda_0)$, the left-hand side of (A.15) is positive by (C1) whereas the right-hand side is negative by (C2), completing the proof. □

A.6 Proof of Proposition 2

In order to simplify the argument, I introduce two technical assumptions. The first assumption requires the firm's expected surplus from a match with a worker who is indifferent between permanent and temporary contracts to be less than the cost of posting a vacancy in the planner's allocation. The other assumption requires the elasticity of the job-finding rate with respect to tightness to be bounded above when evaluated at the planner's allocation. Notice that these two assumptions can be regarded as weaker counterparts of (U1) and (U2), respectively, in Proposition 1. Now, recalling that S_r is independent of labor market tightness (see (15)), I formally state the two assumptions as follows.

(W1) $q(\theta^P)(1-\beta)S_r \leq c$.

(W2) $\left|\theta^P p'(\theta^P)/p(\theta^P)\right| \leq \min\{1, c/S_r\}$.

Proof. Differentiate the planner's objective function (21) with respect to θ and z_r, respectively, to obtain

$$\begin{aligned} 0 &= \int_{\underline{z}}^{z_r} \ell(z)[-u_t'(\theta)] \left[yz - bz - \frac{u_t(\theta)+\theta u_t'(\theta)}{-u_t'(\theta)} c \right] \mathrm{d}z \\ &\quad + \int_{z_r}^{\overline{z}} \ell(z)[-u_p'(\theta)] \left[yz - bz - \delta\kappa - \frac{u_p(\theta)+\theta u_p'(\theta)}{-u_p'(\theta)} c \right] \mathrm{d}z, \end{aligned} \tag{A.16}$$

$$\begin{aligned} 0 &= [1-u_t(\theta)]\, yz_r + u_t(\theta) b z_r - \theta u_t(\theta) c \\ &\quad - [1-u_p(\theta)]\,(yz_r - \delta\kappa) - u_p(\theta) b z_r + \theta u_p(\theta) c. \end{aligned} \tag{A.17}$$

Recall that (21) is uniquely maximized at $(\theta, z_r) = (\theta^P, z_r^P)$ by assumption, implying that (A.16) and (A.17) are satisfied by $(\theta, z_r) = (\theta^P, z_r^P)$. Thus, one can use (A.17) to derive a closed-form solution for z_r^P,

$$z_r^P = \frac{\delta + [1-\mathrm{e}^{-\delta\Lambda}]p(\theta^P)}{\mathrm{e}^{-\delta\Lambda}(y-b)}\kappa - \frac{c}{y-b}\theta^P. \tag{A.18}$$

Meanwhile, the decentralized equilibrium is efficient if and only if $(\theta^*, z_r^*) = (\theta^P, z_r^P)$. Consequently, as a necessary condition for the decentralized equilibrium to be efficient, there must exist a value of $\beta \in (0,1)$ satisfying $z_r^* = z_r^P$, namely,

$$\frac{\delta + [1-\mathrm{e}^{-\delta\Lambda}]p(\theta^P)\beta}{\mathrm{e}^{-\delta\Lambda}(y-b)}\kappa = \frac{\delta + [1-\mathrm{e}^{-\delta\Lambda}]p(\theta^P)}{\mathrm{e}^{-\delta\Lambda}(y-b)}\kappa - \frac{c}{y-b}\theta^P, \tag{A.19}$$

where the left-hand side, coming from (13) with $\theta = \theta^P$ and $r = 0$, is increasing in β while the right-hand side, coming from (A.18), is independent of β. In other words, for $(\theta^*, z_r^*) = (\theta^P, z_r^P)$ to hold true, the value of β needs to be pinned down to

$$\beta_r(\theta^P) := 1 - \left(\frac{\mathrm{e}^{-\delta\Lambda}}{1-\mathrm{e}^{-\delta\Lambda}}\right)\frac{c}{q(\theta^P)\kappa} = 1 - \frac{c}{q(\theta^P)S_r}, \tag{A.20}$$

where the second equality is obtained from (15).

Now, to complete the proof, it suffices to show that, with the value of β given by (A.20), θ^P cannot simultaneously satisfy both the remaining first-order condition for the planner's problem (A.16) and the free-entry condition (19). Recalling $r = 0$ and assuming $z_r^* = z_r^P$, one can rewrite (19) as

$$0 = \int_{\underline{z}}^{z_r^P} \ell(z)u_t(\theta^P) \underbrace{\left[q(\theta^P)(1-\beta)S_t(z;\theta^P) - c\right]}_{<0 \text{ by (W1)}} \mathrm{d}z + \int_{z_r^P}^{\overline{z}} \ell(z)u_p(\theta^P) \underbrace{\left[q(\theta^P)(1-\beta)S_p(z;\theta^P) - c\right]}_{>0 \text{ by (W1)}} \mathrm{d}z, \quad \text{(A.21)}$$

where, thanks to (W1), the first term of the right-hand side is negative whereas the second term is positive. Then, to demonstrate the incompatibility between (A.16) and (A.21), derive, by manipulating the right-hand side of (A.21), the following inequalities:

$$\begin{aligned}
0 &> \int_{\underline{z}}^{z_r^P} \ell(z) \left[u_t(\theta^P) + \theta^P u_t'(\theta^P)\right] q(\theta^P)(1-\beta)S_t(z;\theta^P)\,\mathrm{d}z - \int_{\underline{z}}^{z_r^P} \ell(z) \left[u_t(\theta^P) + \theta^P u_t'(\theta^P)\right] c\,\mathrm{d}z \\
&\quad + \int_{z_r^P}^{\overline{z}} \ell(z) \left[u_p(\theta^P) + \theta^P u_p'(\theta^P)\right] q(\theta^P)(1-\beta)S_p(z;\theta^P)\,\mathrm{d}z - \int_{z_r^P}^{\overline{z}} \ell(z) \left[u_p(\theta^P) + \theta^P u_p'(\theta^P)\right] c\,\mathrm{d}z \\
&> \int_{\underline{z}}^{z_r^P} \ell(z) \left[\frac{\theta^P p'(\theta^P)}{(1-\beta)p(\theta^P)} u_t(\theta^P) + \theta^P u_t'(\theta^P)\right] q(\theta^P)(1-\beta)S_t(z;\theta^P)\,\mathrm{d}z \\
&\quad - \int_{\underline{z}}^{z_r^P} \ell(z) \left[u_t(\theta^P) + \theta^P u_t'(\theta^P)\right] c\,\mathrm{d}z \\
&\quad + \int_{z_r^P}^{\overline{z}} \ell(z) \left[\frac{\theta^P p'(\theta^P)}{(1-\beta)p(\theta^P)} u_p(\theta^P) + \theta^P u_p'(\theta^P)\right] q(\theta^P)(1-\beta)S_p(z;\theta^P)\,\mathrm{d}z \\
&\quad - \int_{z_r^P}^{\overline{z}} \ell(z) \left[u_p(\theta^P) + \theta^P u_p'(\theta^P)\right] c\,\mathrm{d}z,
\end{aligned}$$

where the first inequality[63] holds because $[u_t(\theta^P) + \theta^P u_t'(\theta^P)]/u_t(\theta^P) > [u_p(\theta^P) + \theta^P u_p'(\theta^P)]/u_p(\theta^P) > 0$,[64] and the second inequality holds due to (W2). Notice that the right-hand side of the last inequality is exactly the same as the right-hand side of (A.16),[65] completing the proof by contradiction. □

It is instructive to establish Proposition 2 using an alternative argument that relies on the Hosios (1990) condition. In order for efficiency to be attained (particularly, in random matching and bargaining models), this condition requires that positive and negative matching externalities offset each other. In the benchmark model with random search and Nash bargaining, the Hosios condition simply reads

$$\beta = \beta_h(\theta^P) := 1 - \frac{\theta^P p'(\theta^P)}{p(\theta^P)}. \quad \text{(A.22)}$$

It is interesting to observe that (A.22) makes coefficients of c in (A.16) equal to their counterparts in the free-entry condition. To see this, one can rewrite (A.21) as

[63] One can multiply the first and second terms of (A.21) by $[u_t(\theta^P) + \theta^P u_t'(\theta^P)]/u_t(\theta^P)$ and $[u_p(\theta^P) + \theta^P u_p'(\theta^P)]/u_p(\theta^P)$, respectively, to obtain the right-hand side of the first inequality.

[64] The positivity is implied by (W2).

[65] To establish the equality between them, use both (12) and (A.20) after replacing (θ, z_r) in (A.16) with (θ^P, z_r^P).

$$
\begin{aligned}
0 = &\int_{\underline{z}}^{z_r^P} \ell(z) \left[\frac{(1-\mathrm{e}^{-\delta\Lambda})q(\theta^P)(1-\beta)}{\delta+(1-\mathrm{e}^{-\delta\Lambda})p(\theta^P)\beta} u_t(\theta^P)\right] \left[yz - bz - \frac{\delta+(1-\mathrm{e}^{-\delta\Lambda})p(\theta^P)\beta}{(1-\mathrm{e}^{-\delta\Lambda})q(\theta^P)(1-\beta)} c\right] \mathrm{d}z \\
&+ \int_{z_r^P}^{\overline{z}} \ell(z) \left[\frac{q(\theta^P)(1-\beta)}{\delta+p(\theta^P)\beta} u_p(\theta^P)\right] \left[yz - bz - \delta\kappa - \frac{\delta+p(\theta^P)\beta}{q(\theta^P)(1-\beta)} c\right] \mathrm{d}z,
\end{aligned}
\tag{A.23}
$$

and then verify that $\beta = \beta_h(\theta^P)$ solves both

$$
\begin{aligned}
\frac{u_t(\theta^P) + \theta^P u_t'(\theta^P)}{-u_t'(\theta^P)} &= \frac{\delta+(1-\mathrm{e}^{-\delta\Lambda})p(\theta^P)\beta}{(1-\mathrm{e}^{-\delta\Lambda})q(\theta^P)(1-\beta)}, \\
\frac{u_p(\theta^P) + \theta^P u_p'(\theta^P)}{-u_p'(\theta^P)} &= \frac{\delta+p(\theta^P)\beta}{q(\theta^P)(1-\beta)}.
\end{aligned}
$$

However, it has to be noted that, even with $\beta = \beta_h(\theta^P)$, the integrand of each term in (A.16) does not coincide with its counterpart in (A.23). In other words, $\beta = \beta_h(\theta^P)$ satisfies neither of the following:

$$
\begin{aligned}
-u_t'(\theta^P) &= \frac{(1-\mathrm{e}^{-\delta\Lambda})q(\theta^P)(1-\beta)}{\delta+(1-\mathrm{e}^{-\delta\Lambda})p(\theta^P)\beta} u_t(\theta^P), \\
-u_p'(\theta^P) &= \frac{q(\theta^P)(1-\beta)}{\delta+p(\theta^P)\beta} u_p(\theta^P).
\end{aligned}
$$

In fact, these two equations are solved by

$$
\begin{aligned}
\beta_t(\theta^P) &:= 1 - \left[1-\beta_h(\theta^P)\right] \left[\frac{(1-\mathrm{e}^{-\delta\Lambda})p(\theta^P)+\delta}{(1-\beta_h(\theta^P))(1-\mathrm{e}^{-\delta\Lambda})p(\theta^P)+\delta}\right], \\
\beta_p(\theta^P) &:= 1 - \left[1-\beta_h(\theta^P)\right] \left[\frac{p(\theta^P)+\delta}{(1-\beta_h(\theta^P))p(\theta^P)+\delta}\right],
\end{aligned}
$$

respectively, and it can be easily verified that $\beta_h(\theta^P) < \beta_t(\theta^P) < \beta_p(\theta^P)$. Therefore, additionally noticing that $\beta_h(\theta^P) \neq \beta_r(\theta^P)$ in general, one can conclude not only that the Hosios condition cannot ensure the socially optimal equilibrium, but also that the decentralized equilibrium is not efficient (Proposition 2). Indeed, in the same vein as Davis (2001) and Ljungqvist and Sargent (2012, Chapter 28), there is a "fundamental tension" among the Hosios condition for an efficient supply of vacancies ($\beta = \beta_h(\theta^P)$), the condition for an efficient mix of temporary and permanent workers ($\beta = \beta_t(\theta^P)$ and $\beta = \beta_p(\theta^P)$), and the condition for an efficient choice of contract types ($\beta = \beta_r(\theta^P)$).

Faced with the inefficiency of the decentralized equilibrium, one might wonder how to attain the socially optimal equilibrium. One of the possible ways for achieving efficiency is to allow the bargaining power of type-z workers to depend on their type z. To be specific, the socially optimal equilibrium would be attained if the planner can choose $\beta(z;\theta^P)$ that solves

$$
\begin{aligned}
[-u_t'(\theta)](yz-bz) - \left[u_t(\theta) + \theta u_t'(\theta)\right] c &= u_t(\theta^P) q(\theta^P)[1-\beta(z;\theta^P)] S_t(z;\theta^P) - u_t(\theta^P) c \quad \text{for } z < z_r^P, \\
[-u_p'(\theta)](yz-bz-\delta\kappa) - \left[u_p(\theta) + \theta u_p'(\theta)\right] c &= u_p(\theta^P) q(\theta^P)[1-\beta(z;\theta^P)] S_p(z;\theta^P) - u_p(\theta^P) c \quad \text{for } z > z_r^P,
\end{aligned}
$$

and (A.19) for $z = z_r^P$. Under the assumption that $\beta_r(\theta^P) = \beta_h(\theta^P)$, which ensures $\beta(z;\theta^P)$ to be continuous at $z = z_r^P$, the solution to these equations is given by

$$\beta(z;\theta^P) = \begin{cases} \dfrac{\left[p(\theta^P)u_t(\theta^P) + (1-\mathrm{e}^{-\delta\Lambda})^{-1}\delta\theta^P u_t'(\theta^P)\right](1-\mathrm{e}^{-\delta\Lambda})(yz-bz) + \delta\theta^P u_t'(\theta^P)\theta^P c}{\left[p(\theta^P)u_t(\theta^P) - p(\theta^P)\theta^P u_t'(\theta^P)\right](1-\mathrm{e}^{-\delta\Lambda})(yz-bz) - (1-\mathrm{e}^{-\delta\Lambda})p(\theta^P)\theta^P u_t'(\theta^P)\theta^P c} & z < z_r^P, \\ \dfrac{\left[p(\theta^P)u_p(\theta^P) + \delta\theta^P u_p'(\theta^P)\right](yz-bz-\delta\kappa) + \delta\theta^P u_p'(\theta^P)\theta^P c}{\left[p(\theta^P)u_p(\theta^P) - p(\theta^P)\theta^P u_p'(\theta^P)\right](yz-bz-\delta\kappa) - p(\theta^P)\theta^P u_p'(\theta^P)\theta^P c} & z \geq z_r^P, \end{cases}$$

where $\beta(z;\theta^P)$ can be simplified at $z = z_r^P$, yielding $\beta(z_r^P;\theta^P) = \beta_r(\theta^P) = \beta_h(\theta^P)$. Note that $\beta(z;\theta^P)$ is increasing in z,[66] meaning that the planner prefers workers with high (low) z to have relatively higher (lower) bargaining power. This conversely implies that, in the decentralized equilibrium with the bargaining power constant across workers, a firm is expected to be overcompensated (undercompensated) for its entry cost when matched with a worker with high (low) human capital, resulting in socially nonoptimal entry decisions.

A.7 Deriving the stationary distribution of workers in the extended model

The objective of this subsection is to analytically derive the steady state distribution of the current level of general human capital $z_j \in \mathbb{Z}$ among the unemployed, $\{u(z_j;\theta)\}_{z_j\in\mathbb{Z}}$, which appears in the definition of the stationary equilibrium (38). As previously pointed out, this is a complicated task because of the endogenous dynamics of human capital accumulation. In fact, determining $\{u(z_j;\theta)\}_{z_j\in\mathbb{Z}}$ requires solving for $\{\ell_i^j(\theta), u_i^j(\theta), g_i^j(\theta), s_i^j(\theta)\}_{1\leq i\leq N}^{i\leq j\leq N}$ using the stationary transition equations.[67] Nevertheless, the problem is tractable mainly due to the simplifying assumptions on the dynamics of general human capital.[68] Accordingly, in what follows, I present the transition equations which have to be satisfied in the stationary equilibrium. Since each retired worker is substituted by an unemployed entrant with the same initial level of general human capital, the distribution of initial general human capital across workers is constant over time. Therefore, I describe the stationary transition equations on the basis of an initial level of general human capital $z_0 \in \mathbb{Z}$, rather than its current level $z \in \mathbb{Z}$. Furthermore, $\{u(z_j)\}_{z_j\in\mathbb{Z}}$ must be well-defined out of equilibrium as well as at equilibrium. For this reason, when deriving the stationary transition equations, I consider all possible options[69] that may be chosen by each $z_0 \in \mathbb{Z}$. In what follows, I discuss only simple cases, and readers interested in further details are referred to the online appendix (available upon request).

(H1) *Solving for* $\{\ell_i^j(\theta), u_i^j(\theta), g_i^j(\theta), s_i^j(\theta)\}_{1\leq i\leq N}^{j=i}$.
I first pin down the values of $\ell_i^j(\theta)$, $u_i^j(\theta)$, $g_i^j(\theta)$, and $s_i^j(\theta)$ for the bottom rung of the human capital ladder, which will facilitate the next step associated with the upper rungs of the human capital ladder. Notice that by definition $g_i^i(\theta) = 0$ for all scenarios considered below.

i. *When* $S(z_j) = S_{p,n}(z_j)$.
For a worker with $z_0 = z_i$, both types of human capital do not evolve under the current option. Thus, the argument made for the benchmark model (see Section 2.2) can be borrowed for the

[66] Although not explicitly stated, it is assumed that $\beta(z;\theta^P) \in (0,1)$ for all $z \in \mathbb{Z}$, which is not too restrictive.
[67] See Section 3.2 for the definitions of $\ell_i^j(\theta)$, $u_i^j(\theta)$, $g_i^j(\theta)$, and $s_i^j(\theta)$.
[68] Recall that there is no depreciation of general human capital, and that only one-level upgrade of human capital is allowed during the employment relationship with the current employer.
[69] Namely, $\{(p,n),(p,s),(p,g),(t,n),(t,s),(t,g)\}$, as listed and explored below.

current purpose, implying that $\ell_i^i(\theta) = \ell(z_i)$ and $s_i^i(\theta) = 0$. Furthermore, (16) is extended to

$$[p(\theta) + \rho]\, u_i^i(\theta) = \delta \left[\ell(z_i) - u_i^i(\theta)\right] + \rho \ell(z_i),$$

where the left-hand side is the outflow from unemployment whereas the right-hand side is the inflow into unemployment. Rearranging terms yields the extended counterpart of (17), namely,

$$u_i^i(\theta) = \frac{\delta + \rho}{p(\theta) + \delta + \rho}\ell(z_i). \tag{A.24}$$

ii. *When $S(z_j) = S_{p,s}(z_j)$.*
The level of general human capital does not change under the current option so that $\ell_i^i(\theta) = \ell(z_i)$. Meanwhile, job separation and retirement are not affected by the specific human capital level, and thus, $u_i^i(\theta)$ is determined by (A.24). Lastly, in a stationary equilibrium, the outflow from and inflow into $s_i^i(\theta)$ must balance each other, namely,

$$(\delta + \rho)\, s_i^i(\theta) = \phi_s \left[\ell(z_i) - u_i^i(\theta) - s_i^i(\theta)\right],$$

from which one can arrive at

$$s_i^i(\theta) = \frac{\phi_s}{\delta + \rho + \phi_s}\left[\ell(z_i) - u_i^i(\theta)\right]. \tag{A.25}$$

iii. *When $S(z_j) = S_{t,n}(z_j)$.*
As in the case of $S(z_j) = S_{p,n}(z_j)$, one can recall the argument in the benchmark model to obtain $\ell_i^i(\theta) = \ell(z_i)$, $s_i^i(\theta) = 0$, and

$$u_i^i(\theta) = \frac{\delta + \rho}{[1 - \mathrm{e}^{-(\delta+\rho)\Lambda}]p(\theta) + \delta + \rho}\ell(z_i). \tag{A.26}$$

iv. *When $S(z_j) = S_{t,s}(z_j)$.*
Applying a similar argument to the case of $S(z_j) = S_{p,s}(z_j)$, one can conclude that $\ell_i^i(\theta) = \ell(z_i)$, $u_i^i(\theta)$ is given by (A.26), and

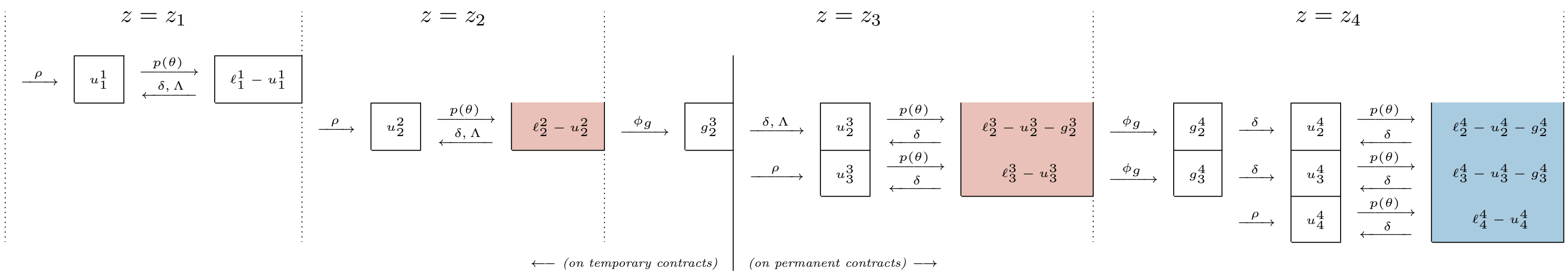
$$s_i^i(\theta) = \frac{\phi_s}{\delta + \rho + \phi_s}\left[\ell(z_i) - u_i^i(\theta)\right] - \frac{\mathrm{e}^{-(\delta+\rho)\Lambda}(1 - \mathrm{e}^{-\phi_s\Lambda})}{\delta + \rho + \phi_s}p(\theta)u_i^i(\theta). \tag{A.27}$$

Notice that (A.27) is obtained from the following balance equation:

$$(\delta + \rho)\, s_i^i(\theta) + \mathrm{e}^{-(\delta+\rho)\Lambda}(1 - \mathrm{e}^{-\phi_s\Lambda})p(\theta)u_i^i(\theta) = \phi_s \left[\ell(z_i) - u_i^i(\theta) - s_i^i(\theta)\right],$$

where the left-hand and right-hand sides represent the outflow from and inflow into $s_i^i(\theta)$, respectively. In particular, the second term in the left-hand side stands for the outflow from $s_i^i(\theta)$ due to the presence of Λ : Among newly-formed (t, s) matches whose flow is equal to $p(\theta)u_i^i(\theta)$, only a fraction $\mathrm{e}^{-(\delta+\rho)\Lambda}(1 - \mathrm{e}^{-\phi_s\Lambda})$ survives until Λ and completes specific training before Λ; however, they are forced to go back to unemployment when their tenure reaches Λ.

Figure 11: A graphical illustration of the unemployment inflows and outflows when $\mathbb{Z} = \{z_1, z_2, z_3, z_4\}$



Notes: The diagram is drawn for the case where $(z_1, z_2, z_3, z_4) = (z_{t,n}^{t,g}, z_{t,g}^{p,g}, z_{p,g}^{p,s}, z_N)$ and $S(z_N) = S_{p,s}(z_N)$. Workers who are on the red areas receive general training, while those on the blue areas receive specific training. The inflows and outflows related to (s_2^4, s_3^4, s_4^4) are omitted for graphical simplicity.

A.8 Welfare properties of the extended model

In this subsection, I formulate the social planner's problem in the extended model. In the environment of the extended model, the planner chooses the labor market tightness $\theta \in (0, \theta)$ to maximize

$$\sum_{z_i \in \mathbb{Z}} \max \left\{ S^*_{p,n}(z_i, \theta), S^*_{p,s}(z_i, \theta), S^*_{p,g}(z_i, \theta), S^*_{t,n}(z_i, \theta), S^*_{t,s}(z_i, \theta), S^*_{t,g}(z_i, \theta) \right\},$$

where

$$\begin{aligned}
S^*_{p,n}(z_i, \theta) &= \left\{ \ell^i_i(\theta) - u^i_i(\theta) \right\} \{y_0 z_i - \delta\kappa\} + u^i_i(\theta) b z_i - \theta u^i_i(\theta) c, \\
S^*_{p,s}(z_i, \theta) &= \left\{ \ell^i_i(\theta) - u^i_i(\theta) - s^i_i(\theta) \right\} \{y_0 z_i - \delta\kappa - \tau_s(z_i) y_0 z_i\} + s^i_i(\theta) \left(y_1 z_i - \delta\kappa \right) + u^i_i(\theta) b z_i - \theta u^i_i(\theta) c, \\
S^*_{p,g}(z_i, \theta) &= \left\{ \ell^i_i(\theta) - u^i_i(\theta) \right\} \{y_0 z_i - \delta\kappa - \tau_g(z_i) y_0 z_i\} + u^i_i(\theta) b z_i - \theta u^i_i(\theta) c + \mathbb{1}_{\{i < |\mathbb{Z}|\}} \times \\
&\quad \left[g^{i+1}_i(\theta) \left(y_0 z_{i+1} - \delta\kappa \right) + \left\{ \sum_{k \geq i+1} \ell^k_{i+1}(\theta) \right\}^{-1} \left\{ \sum_{k \geq i} \ell^k_i(\theta) - \ell^i_i(\theta) - g^{i+1}_i(\theta) \right\} S^*(z_{i+1}, \theta) \right], \\
S^*_{t,n}(z_i, \theta) &= \left\{ \ell^i_i(\theta) - u^i_i(\theta) \right\} y_0 z_i + u^i_i(\theta) b z_i - \theta u^i_i(\theta) c, \\
S^*_{t,s}(z_i, \theta) &= \left\{ \ell^i_i(\theta) - u^i_i(\theta) - s^i_i(\theta) \right\} \{y_0 z_i - \tau_s(z_i) y_0 z_i\} + s^i_i(\theta) y_1 z_i + u^i_i(\theta) b z_i - \theta u^i_i(\theta) c, \\
S^*_{t,g}(z_i, \theta) &= \left\{ \ell^i_i(\theta) - u^i_i(\theta) \right\} \{y_0 z_i - \tau_g(z_i) y_0 z_i\} + u^i_i(\theta) b z_i - \theta u^i_i(\theta) c + \mathbb{1}_{\{i < |\mathbb{Z}|\}} \times \\
&\quad \left[g^{i+1}_i(\theta) y_0 z_{i+1} + \left\{ \sum_{k \geq i+1} \ell^k_{i+1}(\theta) \right\}^{-1} \left\{ \sum_{k \geq i} \ell^k_i(\theta) - \ell^i_i(\theta) - g^{i+1}_i(\theta) \right\} S^*(z_{i+1}, \theta) \right],
\end{aligned}$$

subject to the stationary transition equations described in Appendix A.7.

B Additional Material - Quantitative Part

B.1 Descriptive statistics for the KLIPS

Table 7: Descriptive statistics for the KLIPS-13

	(1) All employees	(2) Permanent	(3) Temporary
Highest level of education			
Primary	609 (18.0)	384 (11.4)	225 (6.7)
Secondary	1,357 (40.2)	1,072 (31.7)	285 (8.4)
Tertiary	1,411 (41.8)	1,311 (38.8)	100 (3.0)
Age			
16 to 29	819 (24.3)	686 (20.3)	133 (3.9)
30 to 55	2,325 (68.9)	1,926 (57.0)	399 (11.8)
56 to 65	233 (6.9)	155 (4.6)	78 (2.3)
Gender			
Male	2,070 (61.3)	1,746 (51.7)	324 (9.6)
Female	1,307 (38.7)	1,021 (30.2)	286 (8.5)
Marital status			
Married	2,195 (65.0)	1,796 (53.2)	399 (11.8)
Not married	1,182 (35.0)	971 (28.8)	211 (6.3)
On-the-job training			
Received	524 (15.5)	478 (14.2)	46 (1.4)
Not received	2,853 (84.5)	2,289 (67.8)	564 (16.7)
Presence of labor union			
Present	681 (20.2)	622 (18.4)	59 (1.8)
Not present	2,696 (79.8)	2,145 (63.5)	551 (16.3)
The total number of employees			
1 to 9	1,004 (29.7)	706 (20.9)	298 (8.8)
10 to 99	1,101 (32.6)	934 (27.7)	167 (5.0)
100 or above	1,272 (37.7)	1,127 (33.4)	145 (4.3)
Occupation			
Managers and professionals	836 (24.8)	776 (23.0)	60 (1.8)
Clerks	745 (22.1)	685 (20.3)	60 (1.8)
Service or sales workers	449 (13.3)	314 (9.3)	135 (4.0)
Elementary occupations	359 (10.6)	206 (6.1)	153 (4.5)
All other occupations	988 (29.3)	786 (23.3)	202 (6.0)
Industry			
Agriculture/forestry/fishing and construction	337 (10.0)	183 (5.4)	154 (4.6)
Extractive and manufacturing	984 (29.1)	884 (26.2)	100 (3.0)
Electricity/gas/water supply	18 (0.5)	17 (0.5)	1 (0.0)
Trade, hotels, and restaurants	633 (18.7)	467 (13.8)	166 (4.9)
Transportation and communication	228 (6.8)	206 (6.1)	22 (0.7)
Financial/insurance/real estate activities	245 (7.3)	219 (6.5)	26 (0.8)
Public service activities	932 (27.6)	791 (23.4)	141 (4.2)

Notes: The table reports the number of employees by personal and job characteristics (with percentages in parentheses), using the 13th wave of KLIPS (KLIPS-13, corresponding to the 2010 survey). The sample size is 3,377, 18.1% (81.9%) of which are classified as temporary (permanent, respectively) employees. Column (1) includes all employees; only permanent and temporary employees are counted in columns (2) and (3), respectively. *On-the-job training* is recorded as "Received" if the individual has received on-the-job training at least once during the past twelve months and "Not received" otherwise. *Occupation* and *Industry* are categorized according to the 6th Korean Standard Classification of Occupations (KSCO) and the 9th Korean Standard Industrial Classification (KSIC), respectively.

B.2 Reduced-form analysis

Table 8: Determinants of temporary employment, 2010 (KLIPS)

	(1)	(2)	(3)	(4)	(5)
Constant	−0.337***	−0.430***	−0.182**	0.089	0.840***
	(0.051)	(0.080)	(0.086)	(0.104)	(0.130)
Highest level of education					
Secondary education	−0.488***	−0.472***	−0.436***	−0.305***	−0.293***
	(0.063)	(0.068)	(0.069)	(0.072)	(0.075)
Tertiary education	−1.133***	−1.126***	−1.019***	−0.686***	−0.734***
	(0.071)	(0.078)	(0.080)	(0.092)	(0.096)
Age					
16 to 29		0.138*	0.139*	0.211***	0.227***
		(0.076)	(0.077)	(0.079)	(0.081)
56 to 65		0.195**	0.226**	0.127	0.088
		(0.094)	(0.095)	(0.097)	(0.103)
Gender					
Female		0.159***	0.119**	0.138**	0.291***
		(0.053)	(0.054)	(0.059)	(0.063)
Marital status					
Married		−0.047	−0.019	0.019	0.011
		(0.065)	(0.066)	(0.067)	(0.070)
Presence of labor union					
Present			−0.294***	−0.218**	−0.069
			(0.089)	(0.091)	(0.096)
The total number of employees					
10 to 99			−0.431***	−0.394***	−0.326***
			(0.064)	(0.066)	(0.069)
100 or above			−0.365***	−0.345***	−0.223***
			(0.074)	(0.076)	(0.079)
Occupation					
Managers and professionals				−0.839***	−0.877***
				(0.106)	(0.112)
Clerks				−0.838***	−0.877***
				(0.106)	(0.110)
Service or sales workers				−0.310***	−0.344***
				(0.097)	(0.104)
All other occupations				−0.453***	−0.428***
				(0.084)	(0.095)
Industry					
Extractive and manufacturing					−1.286***
					(0.097)
Electricity/gas/water supply					−0.897**
					(0.448)
Trade, hotels, and restaurants					−0.838***
					(0.112)
Transportation and communication					−1.287***
					(0.148)
Financial/insurance/real estate activities					−1.093***
					(0.141)
Public service activities					−0.832***
					(0.101)
No. obs.	3,484	3,484	3,484	3,484	3,484
Pseudo-R^2	0.084	0.091	0.116	0.143	0.203

Notes: The table reports estimation results from a probit model whose dependent variable (*Temporary*) is equal to one if the individual holds a temporary contract and zero otherwise. Among all 3,484 dependent workers surveyed in 2010 (corresponding to the 13th wave of KLIPS), 625 workers (17.9%) are classified as temporary employees. All the independent variables used are dummies. The dummies for *Occupation* are relative to "Elementary occupations" while the baseline dummy for *Industry* is "Agriculture/forestry/fishing and construction." The *Occupation* and *Industry* categories are constructed according to the 6th KSCO and the 9th KSIC, respectively. Standard errors are reported in parentheses. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

Table 9: Determinants of receiving on-the-job training, 2010 (KLIPS)

	(1)	(2)	(3)	(4)	(5)
Constant	−0.943***	−1.351***	−1.509***	−1.699***	−2.090***
	(0.028)	(0.080)	(0.107)	(0.075)	(0.133)
Contract type					
Temporary contract	−0.494***	−0.330***	−0.295***	−0.250***	−0.126
	(0.080)	(0.084)	(0.084)	(0.086)	(0.090)
Highest level of education					
Secondary education		0.264***	0.258***		0.188*
		(0.089)	(0.094)		(0.101)
Tertiary education		0.596***	0.603***		0.421***
		(0.088)	(0.094)		(0.102)
Age					
16 to 29			0.044		0.101
			(0.078)		(0.084)
56 to 65			−0.114		−0.097
			(0.127)		(0.133)
Gender					
Female			−0.185***		−0.092
			(0.059)		(0.062)
Marital status					
Married			0.316***		0.240***
			(0.070)		(0.075)
Presence of labor union					
Present				0.217***	0.202***
				(0.074)	(0.075)
The total number of employees					
10 to 99				0.305***	0.281***
				(0.087)	(0.089)
100 or above				0.903***	0.844***
				(0.087)	(0.088)
Sector by ownership					
Public sector				0.289**	0.254**
				(0.117)	(0.118)
Job tenure					
(continuous, in years)				0.019***	0.018***
				(0.005)	(0.005)
No. obs.	3,375	3,375	3,375	3,375	3,375
Pseudo-R^2	0.014	0.036	0.050	0.129	0.143

Notes: The table reports estimation results from a probit model whose dependent variable (*On-the-job training*) takes the value of one if the individual has received on-the-job training at least once during the previous twelve months and zero otherwise. Among all 3,375 dependent workers surveyed in 2010 (corresponding to the 13th wave of KLIPS), 524 workers (15.5%) are classified as those with *On-the-job training* = 1 while 610 workers (18.1%) as temporary employees. All the independent variables are dummies, except for *Job tenure* that is a continuous variable expressed in years. Standard errors are reported in parentheses. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

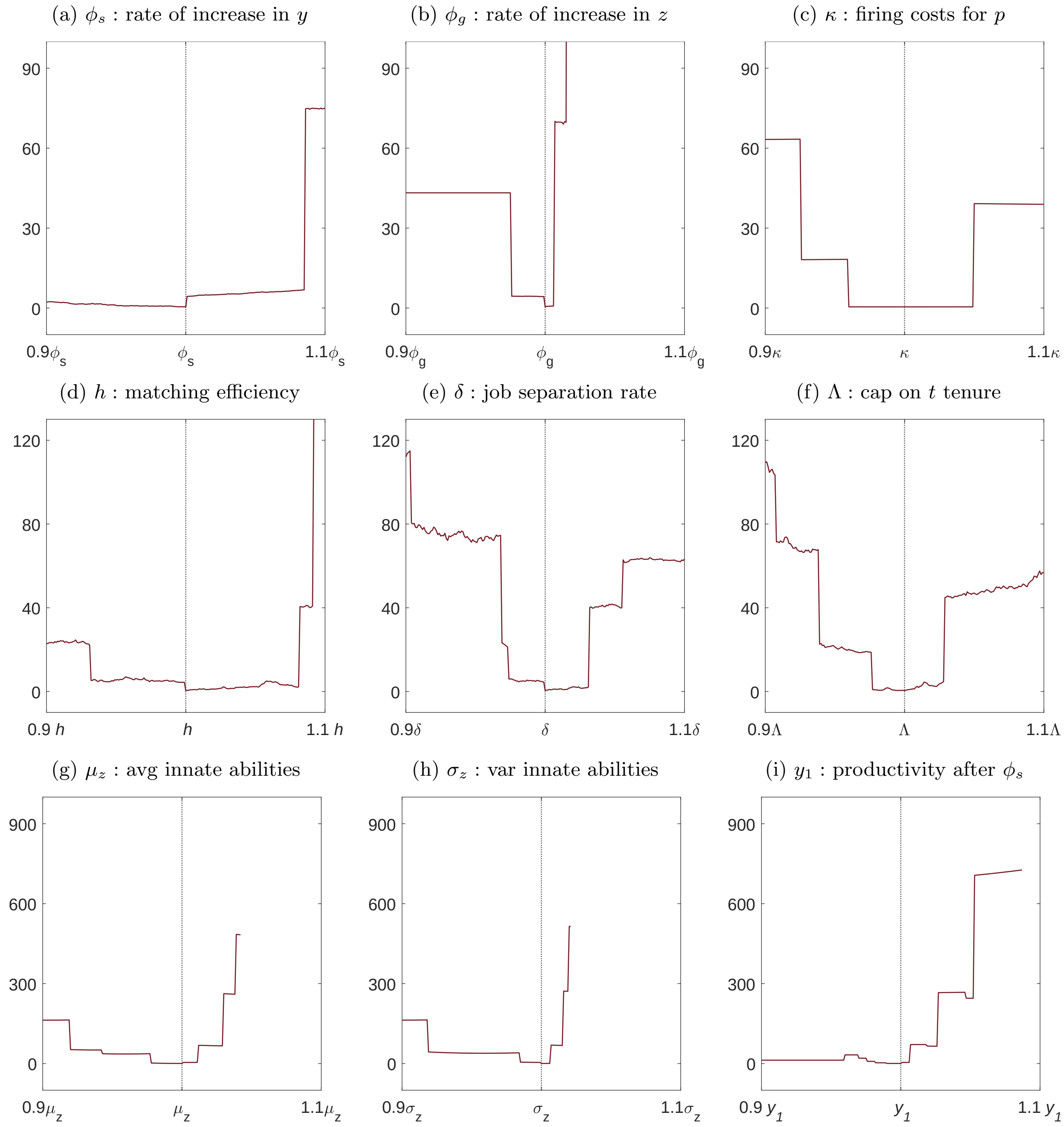
Table 10: Determinants of conversion from temporary to permanent employment, 2010-2016 (KLIPS)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	−1.238***	−0.928***	−0.783***	−0.712***	−0.700***	−0.556***	−0.432***
	(0.092)	(0.086)	(0.089)	(0.096)	(0.106)	(0.116)	(0.136)
Training experience							
(extensive margin)	0.038	0.056	0.055	0.189**	0.255**	0.255**	0.190
	(0.087)	(0.083)	(0.087)	(0.092)	(0.101)	(0.112)	(0.132)
Highest level of education							
Secondary education	0.249***	0.236***	0.242***	0.234***	0.267***	0.266***	0.266***
	(0.068)	(0.062)	(0.064)	(0.069)	(0.075)	(0.084)	(0.099)
Tertiary education	0.353***	0.321***	0.335***	0.500***	0.549***	0.582***	0.787***
	(0.093)	(0.089)	(0.094)	(0.101)	(0.114)	(0.129)	(0.154)
Age							
16 to 29	0.235**	0.293***	0.409***	0.397***	0.636***	0.527***	0.471***
	(0.100)	(0.098)	(0.106)	(0.119)	(0.138)	(0.156)	(0.178)
56 to 65	−0.316***	−0.347***	−0.401***	−0.452***	−0.536***	−0.589***	−0.711***
	(0.078)	(0.072)	(0.078)	(0.087)	(0.101)	(0.119)	(0.150)
Marital status							
Married	−0.073	−0.096	−0.094	−0.173**	−0.164**	−0.208**	−0.171
	(0.063)	(0.060)	(0.064)	(0.069)	(0.077)	(0.088)	(0.105)
Presence of labor union							
Present	0.073	0.109	0.214**	0.260**	0.303**	0.359**	0.470***
	(0.113)	(0.106)	(0.108)	(0.118)	(0.127)	(0.146)	(0.164)
Occupation							
Managers and professionals	0.079	0.216**	0.279**	0.350***	0.383***	0.356**	0.122
	(0.104)	(0.101)	(0.108)	(0.121)	(0.138)	(0.157)	(0.191)
Clerks	−0.003	0.139	0.218*	0.265**	0.373***	0.357**	0.447**
	(0.115)	(0.110)	(0.115)	(0.126)	(0.142)	(0.160)	(0.183)
Service or sales workers	−0.019	0.044	0.066	0.178**	0.267***	0.175	0.074
	(0.078)	(0.075)	(0.079)	(0.087)	(0.097)	(0.110)	(0.131)
All other occupations	0.036	0.106	0.072	0.190**	0.226***	0.143	0.035
	(0.072)	(0.068)	(0.071)	(0.077)	(0.085)	(0.094)	(0.107)
Job tenure							
(continuous, in years)	−0.043***	−0.041***	−0.038***	−0.041***	−0.042***	−0.041***	−0.054***
	(0.007)	(0.006)	(0.006)	(0.007)	(0.008)	(0.009)	(0.011)
No. obs.	4,539	3,739	3,069	2,464	1,965	1,505	1,105
Pseudo-R^2	0.058	0.065	0.079	0.099	0.125	0.124	0.147
Pr(*Temp to perm* = 1)	0.096	0.167	0.215	0.250	0.280	0.298	0.311
Pr(*Training experience* = 1)	0.098	0.099	0.101	0.105	0.106	0.116	0.119

Notes: The table reports estimation results from a probit model whose dependent variable (*Temp to perm*) equals one if the temporary worker in the baseline years (2010 to $2017 - y$, corresponding to the 13th to $(20 - y)$-th waves of KLIPS) achieves permanent employment status y years later, with y being equal to 1 in column (1), 2 in column (2), and so on, and zero otherwise. The variable *Training experience* takes the value of one if the temporary worker has received on-the-job training at least once during the previous three years and zero otherwise. The *Occupation* categories are constructed according to the 6th KSCO, and "Elementary occupations" is used as the baseline dummy. All the independent variables are dummies except for *Job tenure,* a continuous variable expressed in years. Standard errors are reported in parentheses. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

B.3 Additional tables and figures

Figure 12: One-dimensional slices of the objective function for the method of simulated moments



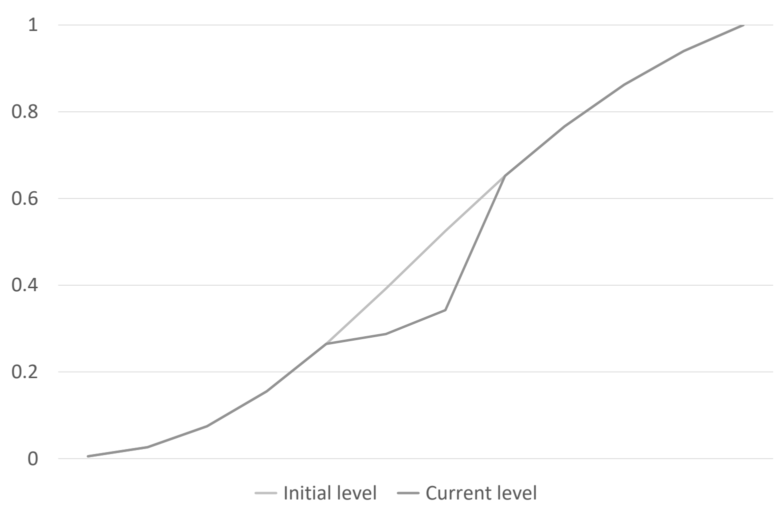
Notes: In each figure, the objective function for estimation (see Section 4.2 for its definition) is evaluated over a range from 10% below to 10% above the estimated value of the parameter of interest, with all the other parameters fixed as estimated. The plot is incomplete in (g)–(i) because the objective function cannot be evaluated on the corresponding interval. The dashed vertical line in the plots indicates where the objective function is minimized; obviously, the line crosses the x-axis at each parameter estimate. The minimized value of the objective function is 0.439.

Table 11: Elasticity of moments with respect to parameters

	ϕ_s	ϕ_g	κ	h	δ	Λ	μ_z	σ_z	y_1	c
S1.	−0.87	0.00	0.00	0.00	0.61	0.00	0.00	0.00	−3.73	0.00
S2.	−0.04	13.33	13.33	−0.05	−5.00	13.33	13.33	13.33	13.33	0.23
S3.	0.34	−0.04	−0.50	0.16	0.62	0.12	0.31	0.22	0.11	0.07
S4.	6.74	13.33	13.33	6.68	2.30	13.33	13.33	13.33	13.33	0.58
S5.	0.01	1.78	1.78	0.01	5.19	1.89	−3.96	−3.90	−16.08	−0.03
J1.	−0.37	−0.22	−0.08	0.01	−0.51	0.04	0.86	0.72	0.88	−0.83
J2.	0.33	0.01	0.05	0.00	−0.69	−0.01	0.42	0.34	0.51	0.14
J3.	−0.51	−0.36	−0.12	−0.36	−0.29	0.44	−0.10	−0.40	0.02	0.24
A1.	0.06	0.20	0.19	0.01	0.28	0.20	0.39	0.39	−0.54	−0.03
A2.	−0.02	−0.01	0.01	−0.02	0.93	0.12	−0.01	−0.17	−1.78	−0.09
A3.	−0.05	−1.16	−1.13	0.00	1.29	−0.54	−1.91	−2.59	−2.88	0.44
T1.	0.47	0.46	0.21	−1.49	−1.45	0.76	4.37	4.18	2.99	−3.57

Notes: The table is obtained by postulating a 7.5 percent decrease in the estimated parameter values. A positive (negative) sign means that a moment moves in the same (opposite) direction as the parameter of interest, that is, the moment decreases (increases) as the parameter of interest decreases. The shaded areas correspond to the heuristic identification argument made in Section 4.2. See Table 2 for descriptions of each moment and each parameter.

Figure 13: Estimated cumulative distribution of general human capital



Notes: The light grey curve represents the estimated cumulative distribution of z_0; the dark grey curve stands for that of z. The area between these two curves indicates that there are some workers who have “upgraded” their general human capital through general training.

Table 12: Scarring effects on wages in counterfactual scenarios

	$\log(wage)$	log(*wage*) components: log of ...					
		z_0	z_τ	y_0	y_τ	$1-\tau$	$wage/Y$
A. *Status quo*							
Constant	2.785	1.112	0.000	1.792	0.116	−0.112	−0.123
First contract type							
Temporary contract	−0.242	−0.348	0.134	-	−0.017	−0.012	0.000
Current contract type							
Temporary contract	−0.499	−0.330	−0.134	-	−0.105	0.061	0.009
Labor market experience							
(continuous, in years)	0.001	−0.000	−0.000	-	0.000	0.001	−0.000
B. Perm. subsidy							
Constant	2.547	0.891	0.041	1.792	0.088	−0.193	−0.072
First contract type							
Temporary contract	-	-	-	-	-	-	-
Current contract type							
Temporary contract	-	-	-	-	-	-	-
Labor market experience							
(continuous, in years)	0.002	−0.000	0.001	-	0.001	0.002	−0.000
C. Train. subsidy							
Constant	3.037	1.362	−0.098	1.792	0.112	−0.018	−0.112
First contract type							
Temporary contract	−0.105	−0.315	0.228	-	−0.030	−0.002	0.013
Current contract type							
Temporary contract	−0.473	−0.362	0.036	-	−0.089	−0.048	−0.010
Labor market experience							
(continuous, in years)	0.001	−0.003	0.004	-	0.000	0.000	−0.000
D. Const. planner							
Constant	3.026	1.486	−0.154	1.792	0.111	−0.103	−0.106
First contract type							
Temporary contract	−0.090	−0.209	0.165	-	−0.029	−0.024	0.007
Current contract type							
Temporary contract	−0.699	−0.453	0.119	-	−0.088	−0.221	−0.057
Labor market experience							
(continuous, in years)	0.003	−0.004	0.005	-	0.000	0.001	−0.000

Notes: Panel A of the table is the same as Table 4. Since there are no temporary workers when permanent jobs are optimally subsidized, the coefficients of *First contract type* and *Current contract type* are not reported in Panel B.

Table 13: Sensitivity analysis with different bargaining weights

		(1) $\beta = 0.3$	(2) $\beta = 0.5$	(3) $\beta = 0.7$
Estimated parameters				
rate of increase in y	ϕ_s	0.685	0.687	0.825
rate of increase in z	ϕ_g	0.320	0.296	0.299
firing costs for p	κ	9.734	8.322	8.024
matching efficiency	h	0.644	0.850	0.684
job separation rate	δ	0.114	0.115	0.117
cap on t tenure	Λ	7.883	7.625	7.641
avg innate abilities	μ_z	7.890	6.421	6.615
var innate abilities	σ_z	0.398	0.443	0.428
productivity after ϕ_s	y_1	6.751	7.020	6.811
vacancy cost	c	11.904	7.080	4.022
Targeted moments				
share of p on training		0.184	0.184	0.160
share of t on training		0.059	0.062	0.065
share of t to t		0.702	0.730	0.710
share of p who were t on training		0.001	0.001	0.001
share of t in the labor force		0.285	0.288	0.286
job-finding rate		0.738	0.789	0.709
job tenure of p		6.546	6.814	6.607
job tenure of t		3.055	3.054	3.032
avg log wage of p		2.731	2.757	2.773
avg log wage of t		2.130	2.100	2.131
avg log wage of new t relative to p		0.615	0.630	0.681
tightness		0.688	0.661	0.479
Welfare properties				
total welfare loss		1.292	1.365	1.653
(% of the planner's output)		(9.04)	(9.42)	(11.43)
··· training of perm		0.740	0.528	0.836
(% of the total loss)		(57.3)	(38.7)	(50.6)
··· training of temp		0.522	0.591	0.581
(% of the total loss)		(40.4)	(43.3)	(35.2)
··· contract type		0.030	0.246	0.235
(% of the total loss)		(2.3)	(18.0)	(14.2)

Notes: Column (2) corresponds to the baseline case reported and discussed in the main text.

Table 14: Comparison of estimation results by gender

		(1) All	(2) Male	(3) Female
Estimated parameters				
rate of increase in y	ϕ_s	0.762	0.753	0.687
rate of increase in z	ϕ_g	0.241	0.187	0.296
firing costs for p	κ	9.153	9.331	8.322
matching efficiency	h	0.946	1.000	0.850
job separation rate	δ	0.090	0.101	0.115
cap on t tenure	Λ	10.969	10.191	7.625
avg innate abilities	μ_z	9.427	8.251	6.421
var innate abilities	σ_z	0.440	0.609	0.443
productivity after ϕ_s	y_1	6.578	6.514	7.020
vacancy cost	c	7.431	8.999	7.080
Targeted moments				
share of p on training		0.146 (0.215)	0.158 (0.230)	0.184 (0.174)
share of t on training		0.088 (0.065)	0.110 (0.073)	0.062 (0.055)
share of t to t		0.757 (0.754)	0.771 (0.784)	0.730 (0.716)
share of p who were t on training		0.002 (0.004)	0.002 (0.005)	0.001 (0.002)
share of t in the labor force		0.188 (0.202)	0.189 (0.163)	0.288 (0.290)
job-finding rate		0.824 (0.833)	0.841 (0.848)	0.789 (0.815)
job tenure of p		7.756 (7.614)	7.532 (8.051)	6.814 (6.453)
job tenure of t		4.483 (4.648)	4.137 (5.997)	3.054 (2.939)
avg log wage of p		3.116 (3.129)	3.304 (3.276)	2.757 (2.739)
avg log wage of t		2.524 (2.474)	2.680 (2.752)	2.100 (2.121)
avg log wage of new t relative to p		0.614 (0.593)	0.611 (0.661)	0.630 (0.661)
tightness		0.581 (0.581)	0.638 (0.581)	0.661 (0.581)
Welfare properties				
total welfare loss		1.645	1.992	1.365
(% of the planner's output)		(7.40)	(7.49)	(9.42)
··· training of perm		0.929	1.204	0.528
(% of the total loss)		(56.5)	(60.4)	(38.7)
··· training of temp		0.380	0.458	0.591
(% of the total loss)		(23.1)	(23.0)	(43.3)
··· contract type		0.337	0.330	0.246
(% of the total loss)		(20.5)	(16.6)	(18.0)

Notes: Column (3) corresponds to the baseline case reported and discussed in the main text. In the middle panel named *Targeted moments,* the corresponding data moments are reported in parentheses.